A STUDY

OF

THE ENERGY DISTRIBUTION OF SCATTERED X-RADIATION.

by

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INTRODUCTION.

(I)

When a substance is irradiated by a beam of X-rays there are emitted, in general, by the substance three distinct types of secondary radiation, known as (1) characteristic radiation, (2) corpuscular radiation and (3) scattered radiation. A study of the first type has considerably extended our knowledge of the structure and behaviour of the atom. A study of the scattered radiation has provided us not only with information concerning the atom, but also with some of our most fundamental knowledge regarding the properties and apparently of the structure of radiation itself. Despite the volume of work, both experimental and theoretical, that had of late appeared in this connection, it was evident that a clear body of experimental facts should be ascertained with certainty. For, there had appeared serious contradictions in experimental results, and fuller knowledge had shown the necessity for consideration of factors hitherto disregarded or neglected.

On the theoretical side too, the conceptions put forward appeared inadequate. It was with the object, if possible, of securing a foundation of fact regarding the distribution /

distribution of scattered radiation that the present research was undertaken.

The phenomena of interference, diffraction and polarisation of X-rays, which have been established beyond dispute, lend strong support to the transverse wave theory of these rays; whereas the phenomena of emission and of absorption of radiation, together with that of photo-electric ejections, receive an easy and natural interpretation according to the quantum theory. General belief, however, is that, in all phenomena concerned with the propagation outside the source, X-radiation behaves as waves; while in all those concerned with the production and transformation of the radiation, it is of the nature of energy quanta.

The origin of the Scattered X-rays, according to classical electrodynamics, is to be traced to the forced vibration into which the electrons inside the scattering material are set by the field of the periodic electric vector in the incident wave; the vibrating electron should, therefore, send out a radiation of the same wave length as that of the primary beam. This is the scattered radiation and it should, according to this theory, be of the same "hardness" or absorbability as that of the primary beam.

Experimental evidence, on the contrary, revealed that the scattered X-rays are - at least usually - more absorbable than /

than the primary. This fact Compton ascribed to an increase in the wavelength of the scattered rays; whereas Barkla associated it, in the case of a heterogeneous radiation, with a more general phenomenon - the J-phenomenon.

In an attempt to find an explanation for the softening of the scattered X-rays, Compton made the drastic assumption that the photon collides with the free, scattering electron in the manner of a particle in ordinary dynamics. A part of the energy and momentum of the incident photon is transferred to the electron, resulting in the latter's recoil with a certain amount of energy; while the incident photon rebounds with reduced energy and hence reduced frequency. This rebounding photon is the scattered X-ray in question, possessing an increased wavelength. The increase in wavelength, calculated on this theory, is given by

$$d\lambda = \frac{2h}{mc} \sin^2 \frac{\phi}{2} = .0484 \sin^2 \frac{\phi}{2} \text{ Å.U.}$$

where '$d\lambda$' is the change in wavelength, 'h' is Planck's constant, 'm' the mass of the electron, 'c' the velocity of light, and 'ϕ' the angle between the scattered and incident rays.

If the electron struck is not free, but is held firmly inside the atom, as in the case of heavy atoms, no recoil can take place, and scattered quantum is unmodified. Again, the degree of modification, i.e. the proportion of modified to the unmodified radiation, depends on the frequency /

frequency of the incident photon. In the case of hard γ-rays, the scattered radiation is almost completely modified, but the visible radiation, on the other hand, is left, after scattering, completely unmodified. In the region of X-rays, the radiation is partly modified and partly unmodified; the proportion depending on the incident wavelength as well as on the nature of the scattering substance. The Compton change of wavelength is relatively more important for atoms of low atomic number.

The energy distribution of scattered X-radiation has been a subject of investigation for a long time past, and many theoretical as well as experimental investigations have been conducted, without complete success, in revealing the nature of the fundamental processes occurring in X-ray scattering.

Barkla was the first to show, on the basis of the classical theory of scattering formulated by Sir J. J. Thomson, that, for an unpolarised X-beam, the intensity of the scattered rays should follow the law, $I_\phi = I_{90}(1+\cos^2\phi)$ where I_ϕ and I_{90} are respectively the scattered intensities for angles ϕ and 90°, measured from the direction of the incident ray.

Sir J. J. Thomson (Conduction of electricity through gases, 2nd Edition) worked out from principles of electromagnetic theory a full expression for the intensity of radiation scattered by a single "free" electron, assuming,

of /

of course, that the primary beam is unpolarised. This expression can be put down as

$$I_s = \frac{I}{2} \cdot \frac{e^4}{r^2 m^2 c^4} (1 + \cos^2 \phi),$$

where I_s refers to the intensity of the scattered radiation emitted by a single free electron, I to the intensity of the incident beam; e is the electronic charge, m the mass of the electron, r the distance from the vibrating electron of the point at which the scattered intensity is calculated. Other quantities have the same meaning as before.

If there are Z electrons inside the atom, and if they are very closely packed, they can be regarded as a single scattering unit of charge 'Ze' and mass 'Zm', so that by substituting these values in the above expression, the scattered intensity due to an atom is found to be Z^2 times that due to a single electron. If, on the other hand, the distances between the electrons inside the atom are large compared to the wavelength of the incident beam, no definite phase relation will possibly exist between rays scattered by different electrons, and the total intensity scattered by the atom is Z times that arising from a single electron. Lastly, if the electronic distances be comparable with the incident wavelength, interference, both 'destructive and constructive', may occur between rays scattered by different electrons.

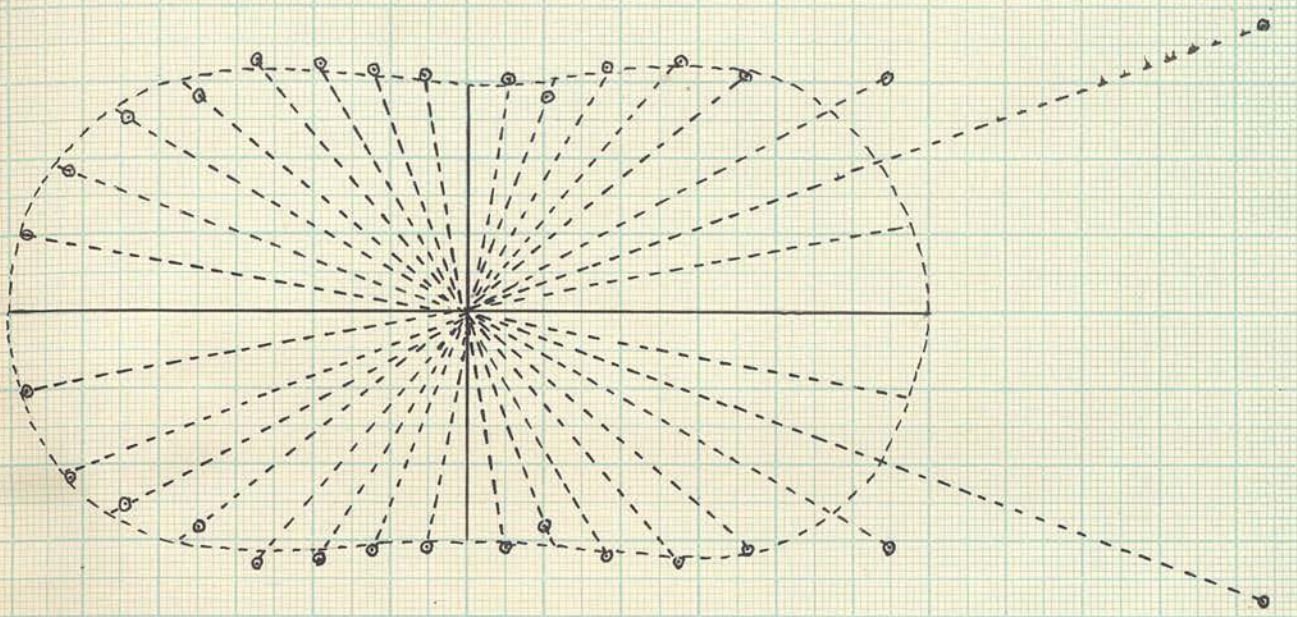
The first experimental test of the above simple classical theory was taken up, as early as 1908, by Barkla himself /

himself, (Phil. Mag. Feb 1908), who showed that the ratio I_{170^o}/I_{90^o} fell down from about 2 to 1.5 with increasing hardness of the incident beam. Stress was therefore laid by the author of the experiment on the use of a very soft radiation as an essential condition for realising classical theoretical results most completely.

A more detailed test was pursued in 1911, by Barkla in collaboration with Ayres (Phil. Mag. Feb 1911). The results of their researches are illustrated in fig 1.(a), where the broken curve denotes the distribution, according to the simple classical theory, and the small circles, the experimental values. The length of the radius vector represents the corresponding scattered intensity, taking the intensity scattered in the direction $\phi = 90^o$ as unity. They showed that, for a carbon radiator, and for a soft incident beam, the agreement between theory and experiment was good for all angles, except for values of ϕ less than 30^o, in which directions the relative scattered intensity "I_ϕ/I_{90^o}" was far in excess of that given by $(1 + \cos^2\phi)$. This excess has since been known as "Excess scattering". No correction was made in these experiments for the difference in absorbability of the scattered rays in different directions. This correction, however, would have been small for such a soft radiation as that used by them.

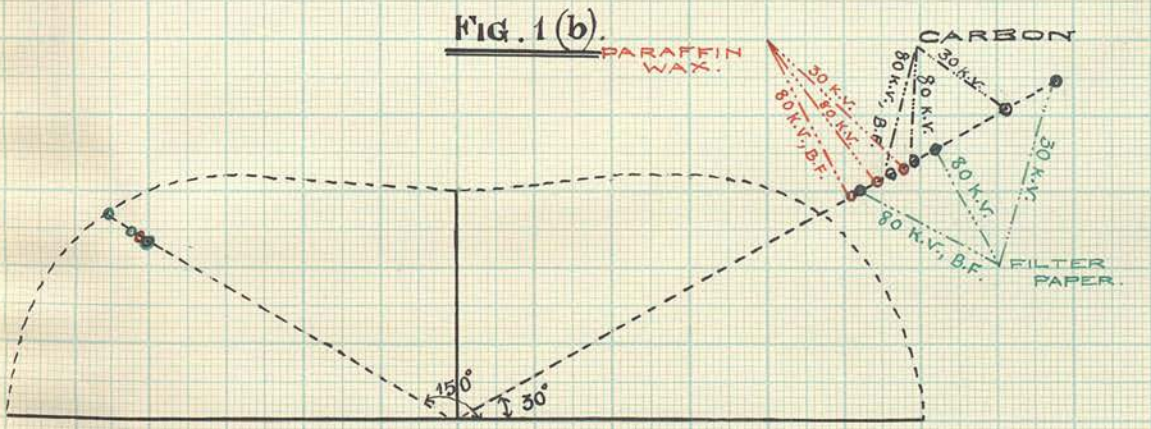
The next work in this connection was that of Owen (Camb. /

Fig. 1 (a).



Barkla & Ayres.

Fig. 1 (b).



Khubchandani.

(Camb. Phil. Soc. Proc.16, 1911), who, by employing thick and thin radiators of filter paper, and radiations of different hardnesses (spark gaps 7 cm., 4.5cm., and 2.5 cm.) observed that the distribution on the incident side, in all cases, was in accordance with the classical theory; it was only in the case of a thin scatterer with the hardest primary radiation that he got the classical distribution fulfilled for the emergent side too. For thick radiators, on the other hand, the relative scattered intensity in the forward direction was in excess, and seemed to increase with the softness of the primary beam. No correction was applied for polarisation of the incident beam or for the Compton change of wavelength. The excess scattering in the forward direction, Owen explained as due to the softer rays in the primary beam being irregularly refracted and subsequently diffused. This effect was considered to be superposed on the true scattering, and to account for the excess.

In the same year, Crowther too, (Proc. Roy. Soc. A.Vol 85 1911), published a description of his investigation on intensity distribution of rays scattered from a thin aluminium sheet. He also found a marked preponderence in the forward direction. The results of his experiment he put thus: "It seems, therefore, that while the maximum in the forward direction is greater than is required by the theory, the maximum in the reverse direction is somewhat less." . . . . . "The distribution of the radiation does not seem /

seem to depend upon the thickness of the radiator, though it does, to some extent, upon the material, the eccentricity being rather less for paper than for aluminium."

About a decade later Hewlett (Phys. Rev. Pp 688, Dec. 1922) studied, by an ionisation method, the scattered intensity for an angular range 2° to 165°, using an approximately, or at any rate a much more homogeneous radiation, ($\lambda = .7$ Å approx.). The substances examined were solids, - carbon (diamond and graphite) and metallic lithium; liquids - benzene, mesitylene and octane. For solids the scattering curves depicted a number of maxima and minima, explained as due to interference; while for liquids there was only one maximum, with indications of others unresolved; this suggested a crystal structure for the solids and liquids in question. Besides, for very small angles, the scattering was zero for solids, and approached zero for liquids.

Shortly after this, newer conceptions about the relation between Waves and Quanta began to develop. Breit (Phys. Rev. 27, 362, 1926), reasoning from the Correspondance Principle of Bohr, arrived at an expression for the intensity distribution of the scattered radiation which was correlated with the wavelength and would account for an excess in the forward direction. This expression can be written as

$$I_{\phi} = I_s\left(1+\frac{h\nu}{mc^2}\ vers\ \phi\right)^{-3}$$

where /

where I_s denotes the classical scattered intensity due to a single free electron, ν the frequency of the primary beam, and all other symbols have thesame meaning as before. Dirac (Proc. Roy. Soc. A.A. CXI, Page 405, 1926), derived the same formula on the basis of quantum dynamics of Heisenberg. The same expression was deduced on the principle of the de Broglie-Schrödinger Wave-theory, by Born and Waller (Phil. Mag 4, 1927, 1928), as well as by Gordon (Zeits. f. Physik 39, 117, 1926). Klein and Nishina (Zeits. f. Physik 52, 582, 1928), on the other hand, obtained a slightly modified scattering function, on the hypothesis of electron-spin, and making use of the relativistic quantum dynamics of Dirac. The deviations of the two above formulae are of the order of $(\frac{h\nu}{mc^2})^2$, whereas the Breit-Dirac expression differs from the classical Thomson formula by quantities of the order of $(\frac{h\nu}{mc^2})$.

Another scattering function of some importance was the outcome of Compton's (Phys. Rev. 35, 925, 1930) theoretical investigation on X-ray scattering. This function reads as

$$I_\phi = I_s\left\{F^2 + \left(Z - \frac{F^2}{Z}\right)\left(1 + \frac{h\nu}{mc^2} \text{vers}\,\phi\right)^{-3}\right\}$$

where $F = \int_0^\infty U(r) \frac{\sin Kr}{Kr}\, dr$, $K = \frac{4\pi}{\lambda} \sin \phi/2$.

The quantity U(r) within the integral, means the radial charge-density of the atom measured in electrons per unit distance. The other quantities have the same meaning as before. In this calculation, electrons are assumed to have a /

a random orientation within the atom and an arbitrary radial distribution. This expression, it may be noted, is the classical expression multiplied by the factor within the curled bracket. The Fourier-integral F admits of evaluation by observing "I " for different angles and wavelengths. A comparison of this formula with Wentzel's theory of X-ray scattering makes it clear that, in order to get a more accurate scattering function, a certain correction factor needs to be introduced. It may be pointed out here, that the function $\psi\psi^*$, in the language of wave-mechanics, which is to be interpreted as the probability of occurrence of an electron, may be regarded as a true measure of the total intensity of the scattered X-rays. Woo(Phys. Rev. 38, 6, 1931) subsequently introduced a correction term e^{-2M} in the expression of Compton, to take account of the effect of temperature on scattering. In this, he was guided by theoretical works (related to scattering by a dynamic atom) of Raman (Ind. J. Phys. 3, 357, 1928) and of A. H. Compton (Phys. Rev. 35, 925, 1930).

The above theoretical speculations, founded on the new wave mechanical conception, gave a fresh incentive to re-investigate the subject of the intensity distribution of scattered X-rays, in all its bearings.

The year 1931 saw Jauncey and Harvey (Phys. Rev. 37, 1203, 1931) reporting very good agreement between scattering from paraffin and the Dirac-theory, near 90°, at a wavelength of /

of about .3Å . Similar agreement was also obtained for a rocksalt crystal at wavelengths of .7Å and .4Å.

In the same year, Coven (Phys. Rev. 38, 1424, 1931) tried the same experiment, using as scatterers, aluminium, paraffin, lead and copper, between the limits 30° and 120°. The scattering from paraffin and aluminium was studied for a radiation of an effective wavelength 1.32 Å , while that from copper and lead was for .27 Å. For the angular range 60° to 120°, paraffin displayed satisfactory agreement with the Dirac-theory; whereas, for the region 30° to 60°, the experimental values were below the theoretical.

The next year, Chylinski (Phys. Rev. pp 42, 153. Oct. 1932) determined experimentally, for various solids, the distribution of the intensity of scattering for angles from 10° to 105°, making correction for the Compton-change in absorbability. Comparison of experimental results for paraffin with predictions of Breit-Dirac theory showed a distinct excess for the experimental values in both the forward and backward directions.

The most recent work in this connection, is that of Backhurst* (Phil. Mag. Feb. 1934) who, using homogeneous wavelengths of .395 Å and .31 Å, and applying corrections for polarisation and the Compton-change of wavelength, found for /

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* Published when the work undertaken in Edinburgh was in progress.

for beryllium the intensity distribution to be in keeping with the Dirac-theory, from 40° to 150°; but for the angle 30°, the observed value was in excess. Measurements were also made with paraffin, water, benzene, turpentine, alcohol and benzophenone. At angles greater than 60°, it was found that the relative intensities were within 2 p.c. the same for all these scatterers as for beryllium. Only two wavelengths, however, were experimented upon and these were only slightly separated. They could not provide an adequate test of the influence of a change of wavelength.

(II)

The conflicting character of the experimental results of different authors (as narrated above), together with the fact that, as yet, there seemed to have been published no regular and systematic work on the dependence of X-ray scattering on the incident wavelength, in the light of the new quantum mechanics, called for a fresh and systematic investigation on the behaviour of a heterogeneous complex beam of X-rays, with respect to the distribution of the scattered intensity. It was evidently desirable to put to experimental test, the validity of the functions, proposed as above, in the case of a heterogeneous X-beam. (The use of a heterogeneous beam enormously increases the range of experimental conditions possible in a single experimental research. It also provides those conditions under which there has been observed what is known as the J-phenomenon, which seems to completely break away from the laws found for the homogeneous constituent radiations.) Such an investigation was undertaken in 1933, in this laboratory, by S. G. Khubchandani, who concentrated his attention mainly on the effect produced by the variation of the average hardness or frequency of the incident primary beam on the angular distribution of the scattered energy. He further studied the influence, on this distribution, of

(1) the material (composition) of the scattering substance,

(2) the thickness of the scatterer,

(3) the /

(3) the source of excitation of the X-radiation, and the procedure consisted in determining the ratio of the scattered intensities, along directions making an angle ϕ and 90° respectively, with the primary beam. Two directions were chosen, one forward and the other backward, with respect to the direction of incidence: they were defined by $\phi = 30^\circ$ and $\phi = 150^\circ$. The X-ray tube used was a watercooled, hot-filament Müller tube, and the radiations were of three different hardnesses corresponding to the voltages (1) 30 k.v., (2) 80 k.v., (3) 80 k.v., - filtered by a thickness .5 cm. of aluminium. The voltages refer to the peak-values and were obtained from a transformer. The different scattering substances employed were (1) Paraffin wax, (2) Carbon, and (3) Filter paper. Corrections (to be explained later on) were applied to the observed values of I_ϕ/I_{90}, on account of (i) polarisation in the incident beam (ii) variation in the ionising power of the radiation scattered in different directions, owing to dependence of wavelength on ϕ (iii) obliquity of some of the radiation entering the ionisation chamber.

His results obtained for a heterogeneous X-radiation excited by a transformer as a source of H.T., are collected in Table 1, which includes, for the purpose of comparison, the corresponding values to be expected on the theory of Dirac /

TABLE I.

Results of Khubchandani's Experiments.

Scattering substance.	Exciting Volt in k.v. (peak)	$(\bar{\mu}/\rho)_{Al.}$*	Equiv. λ Å.U.	I_{30}/I_{90} corrected	I_{30}/I_{90} (Dirac)	I_{150}/I_{90} corrected	I_{150}/I_{90} (Dirac)
Paraffin	30	5.6	.7	2.285	1.90	1.62	1.60
Wax	80	1.88	.49	2.075	1.98	1.59	1.55
.62 cm thick	80 (B.F.)	.71	.34	2.005	2.08	1.61	1.47
Carbon	30	5.6	.7	2.75	1.90	1.58	1.60
.9 cm thick	80	1.88	.49	2.29	1.98	1.58	1.55
	80 (B.F.)	.71	.34	2.18	2.08	1.62	1.47
Filter paper 24 sheets, superficial density .1526 gm/cm^2	30	5.6	.7	3.005	1.90	1.57	1.60
	80	1.88	.49	2.38	1.98	1.65	1.55
	80 (B.F.)	.71	.34	2.01	2.08	1.75	1.47

* The meaning of $(\bar{\mu}/\rho)_{Al.}$ has been explained on Page 29

Dirac. These results are also illustrated in fig 1 (b). From the above table, it will be observed that:

(a) In the case of I_{30}/I_{90},

(1) for all scatterers (paraffin wax, carbon and filter paper) the results do not agree with either the simple classical theory of Sir J. J. Thomson or the more recent theory of Dirac. The ratio is far greater than the classical value 1.75 and increases as the wavelength increases, which is contrary also to Dirac's theory. The value I_{30}/I_{90} seems to approach the limiting classical value 1.75 for very hard radiation;

(2) the excess scattering for a particular radiation is different for different scattering substances. Filter paper shows a great sensitiveness to change in wavelength.

(b) In the case of I_{150}/I_{90},

(1) for paraffin and carbon the ratios are considerably lower than the classical value 1.75, but show satisfactory agreement with Dirac's theory for radiations excited at 30 k.v. and 80 k.v. But for the filtered radiation at 80 k.v.(B.F.), the ratios are about 10% higher than the Dirac's theoretical value. Moreover, the ratios show little variation with wavelength.

(2) But for filter paper, only the radiation of longer wavelength (at 30 k.v.) is in agreement with Dirac's /

Dirac's value. (This agreement was overlooked by the author of the experiment). It is important and interesting to note how I_{150}/I_{90} increases, contrary to Dirac's theory, with an increase in the frequency of the radiation, approaching and finally reaching the classical value 1.75. Filter paper thus appears to be the only substance for which the classical results are realised for a very hard radiation.

As regards the effect of thickness of the scatterer, experiments performed with different thickness of paraffin wax, showed only a very slight difference - so slight in fact, that it could not be conclusively decided whether this difference was a genuine one or was due to experimental error. The effect of thickness in this case may, therefore, be ignored.

Experiments were also performed in order to study if the ratio of the observed intensities I_{ϕ}/I_{90} was affected by a change of the H.T. source, keeping, of course, the exciting peak voltage the same. An induction coil was substituted for the transformer, and it was found that, for the same peak voltage, the results were practically identical in the two cases.

These extremely interesting results demanded an extension of this work with a view to obtain further information regarding the fundamental processes occurring in the phenomenon of scattering. It is at once evident that /

that further experiments had to be performed to ascertain

(1) whether the excess scattering with its principal features as observed in the region of $\phi = 30^{\circ}$, were in any way exceptional, i.e., whether they were affected by any special relationship of wavelength to crystal or atomic structure;

(2) whether the characteristics of scattering such as are observed for filter paper, Carbon and Paraffin wax, are manifested by other scatterers also;

(3) (a) whether, for $\phi = 30^{\circ}$, the ratio I_{ϕ}/I_{90} could, by progressively increasing the hardness of the incident beam, be made to approach further and finally reach the simple classical value 1.75, or even to surpass this limit.

(b) What are the determining factors - either in the chemical constitution of the scatterer or in the physical process of scattering - that are responsible for the observed differences in the excess scattering for different scatterers, even though the primary radiation in each case be the same?

(c) Whether there is any simple law that connects the excess scattering with any of the known or measurable attributes of the primary beam, such as the effective wavelength.

(4) Whether /

(4) Whether, for $\phi = 150^{\circ}$,

(a) The value of the ratio I_{ϕ} / I_{90}, for filter paper, could be made to exceed the classical value 1.75, by further hardening the incident radiation; or is 1.75 a limiting value?

(b) What happens to the ratio I_{ϕ} / I_{90} for carbon and paraffin wax by increasing the hardness of the incident beam beyond $(\bar{\frac{\mu}{\rho}})_{Al.} = .71$. It is not quite clear that, while for radiations at 30 k.v. and 80 k.v., the value of I_{150}/I_{90} (in the case of carbon and paraffin wax) agrees with Dirac's value, for the radiation at 80 k.v. filtered before incidence, it does not do so. This point deserved further investigation.

These are some of the relevent questions which awaited answer. The present work was, therefore, undertaken with the purpose of obtaining further light on the above questions as far as experimental circumstances permitted. In the present paper an attempt has been made, not only to reinvestigate with a view to confirmation or otherwise, some of the results obtained by the previous worker, but also to make further observations in the region of shorter wavelengths and to study, with a view to generalisation, the excess scattering in another direction, viz $\phi = 20^{\circ}$, and also for another radiator, viz an aluminium sheet. Radiations made progressively/

progressively harder, ranging from an effective wavelength .77 Å.U. to .225 Å.U., have been employed and conclusions deduced. Further, different elementary radiators such as Carbon, Aluminium, Sulphur, etc., have been tried, and the behaviour of the radiations scattered by them, for the same incident beam, studied.

Radiations of various average frequencies were obtained by first varying the applied high tension, by steps, as far as the maximum potential allowed (about 100 k.v.) for the particular tube, and then onward, by progressively filtering the radiation at this maximum voltage, till the intensity reached a minimum workable limit.

The reduced scattered intensity was compensated - in some cases - by having fairly thick radiators. This method, of course, was practicable only when the radiator was a poor absorber, such as paraffin wax, carbon and filter paper; but was not so when it was a good absorber such as aluminium, sulphur, etc.

The employment of a thick radiator might entail inequalities of absorption of the scattered beams proceeding in the two directions concerned, within the radiator. This difficulty was easily eliminated by using the method of Barkla and Ayres (Phil. Mag. Feb. 1911) which requires the radiator to be placed so that the normal to its surface makes equal angles with the two directions studied (ie. $\phi = \phi$ and $\phi = 90$). In addition /

addition, the fact that thin and thick radiators of paraffin wax, placed in this manner, yielded practically identical results in Khubchandani's experiments (which has been later confirmed in our experiments for filter paper too; see page 102) removes causes for all other objections that may be raised against using a thick radiator of paraffin wax or filter paper.

EXPERIMENTAL.

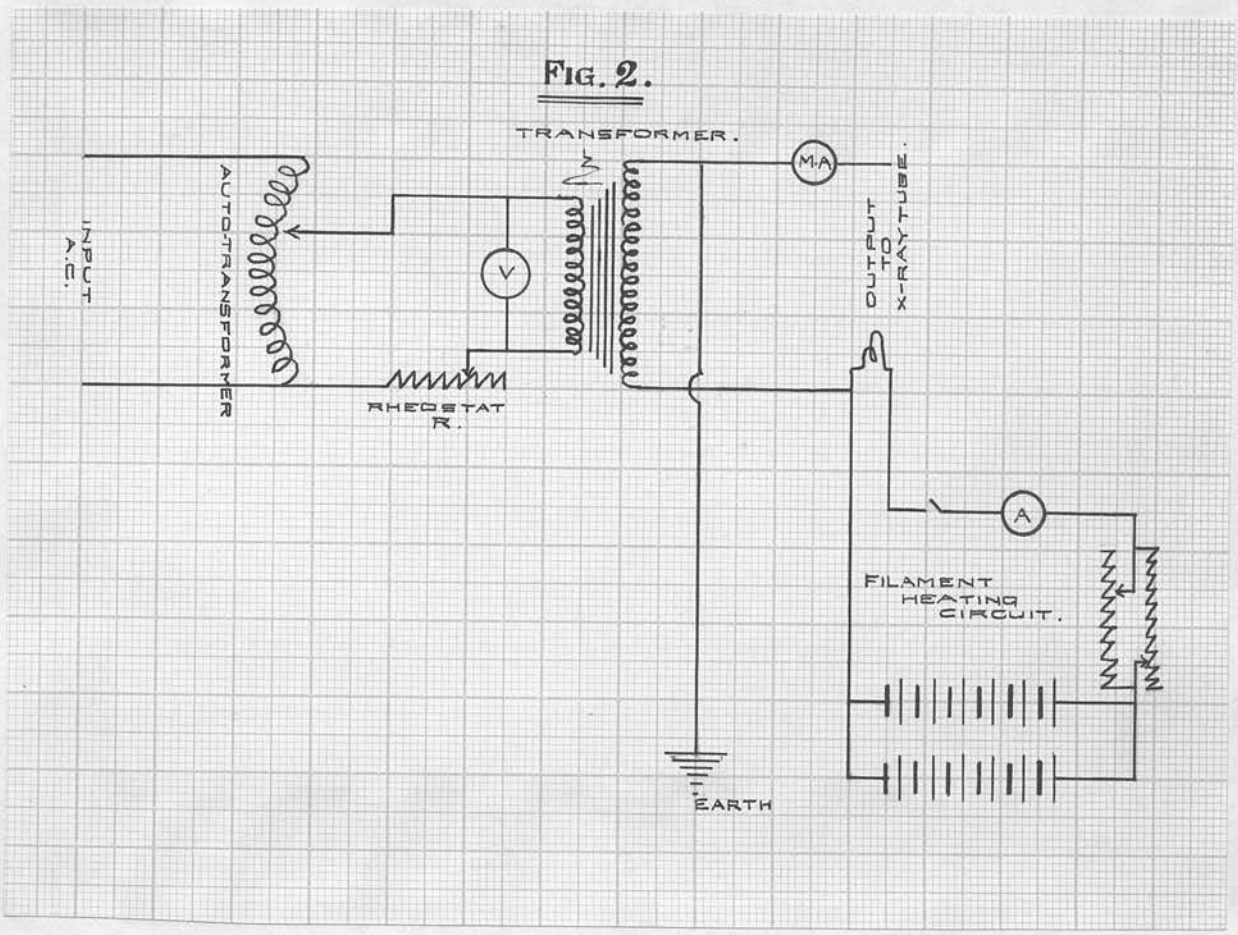
1. Description of the Apparatus.

The apparatus used in the present investigation was essentially the same as that used by the previous worker, Khubchandani, with one or two modifications made in order to increase thefacility and improve the condition of working. It consisted principally of the following parts.

(1) The X-ray tube.

The tube was a water-cooled, hot cathode, self-rectified one, of the Müller type. The anti-cathode was made of tungsten. The tube was supported on a specially constructed wooden stand which could be rotated about a horizontal axis passing through the anticathode spot (where the cathode stream impinges on the anticathode) and perpendicular to the direction of the cathode stream, so that, by turning the stand through a right angle, the cathode stream could be made horizontal or vertical as desired. This horizontal axis of rotation was also the central ray in the primary beam. The tube could be kept fixed in either of these positions by means of a suitable clamping device. The tube and supports were housed inside a wooden box (size 120cm. x 106 cm. x 105 cm.) lined outside with lead sheet (about /

FIG. 2.


TRANSFORMER.
INPUT A.C.
AUTO-TRANSFORMER
V
RHEOSTAT R.
M.A
OUTPUT TO X-RAY TUBE.
A
FILAMENT HEATING CIRCUIT.
EARTH

(about 3 mm. thick) properly earthed. A cylindrical lead tube T (length 14 cm., diameter 2.5 cm.) as shewn in fig 3 (a), served as the outlet for the primary beam of X-rays, the passage of which was controlled by means of a lead shutter S, worked by a string passing over small pulleys.

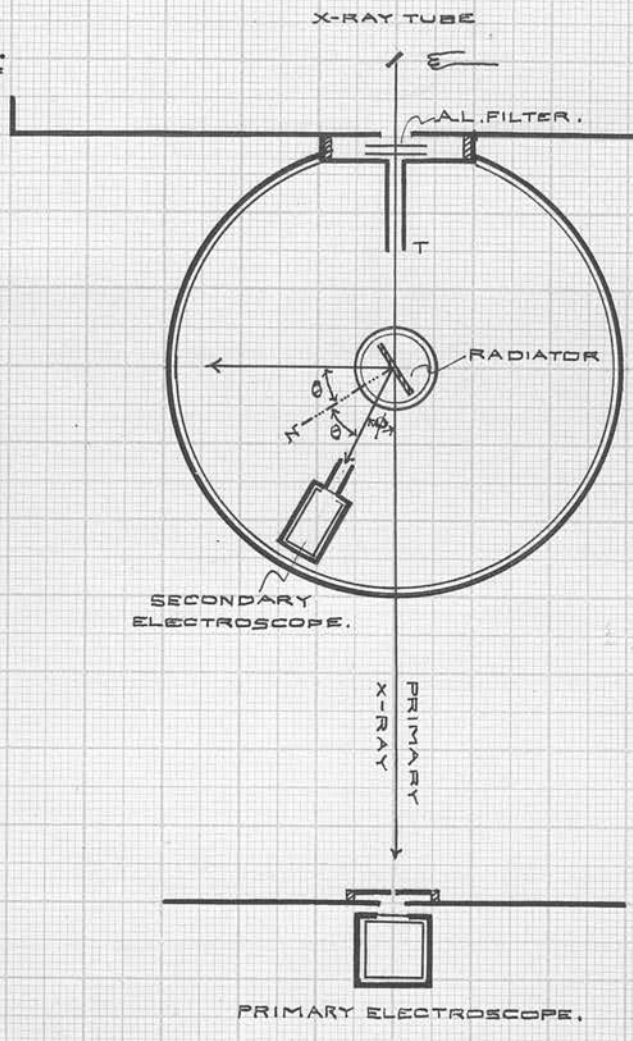
(2) The filament circuit. (Fig. 2.)

The hot-filament cathode was heated to incandescence by an electric current, about 3 amps, derived from a battery of 12 storage cells (of the Exide type, discharging capacity, 105 ampere-hours). These cells were arranged in two series, each containing six and the two series were connected in parallel. For adjusting the filament current to the appropriate value, two variable resistances - 12 ohms and 60 ohms - were connected in parallel to each other, the whole system being in series with the filament. The battery of cells was placed on a well-insulated platform and the resistances were regulated by hand through an ebonite rod. An ammeter in series with the filament indicated the current flowing through the latter. The tube current could be controlled by regulating the filament current and was kept constant during the experiment.

(3) The high-tension circuit. (Fig. 2.)

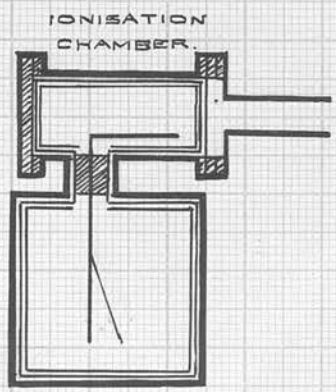
The high tension, for the generation of X-rays, was derived from a transformer and the peak value, measured by the spark gap between two 10 inch spheres. The leads from the A.C. mains, (230 volts $\sim$) were first taken to an auto-transformer, to which a variable rheostat 'R' was attached. From this, a suitable value of the primary voltage /

Fig. 3(a).



X-RAY SPECTROMETER.

Fig. 3(b).



voltage for excitation of the transformer was tapped, and fine adjustments were made with the help of resistance R. The actual primary excitation voltage could be read from a Voltmeter 'V', and the tube current, from a milli-ammeter 'M.A.', connected as shewn in fig. 2. The voltmeter, the milliammeter, as also the tapping arrangements and the exposure switch, were all fixed on a switch-board regulator of "Sunic" type, manufactured by Watson & Co., London.

(4) Ionisation chamber and electroscopes. (Fig 3 (b)).

The electroscopes for measuring the intensity of X-rays were of the usual gold-leaf type. Two electroscopes were used, one to measure or standardize the primary beam, and the other to measure the scattered secondary beam. The latter, on account of the comparatively feeble intensity of the scattered beam, was used in conjunction with a specially constructed ionisation chamber, filled with the more highly ionisable gas SO_2. The ionisation chamber was a brass cylinder (size 7.4 cm. long, 3.2 cm. diameter), provided with a thin aluminium window, and lined inside with a thin aluminium foil (.1 mm. thick) and four-fold filter paper. The aluminium lining absorbed the characteristic radiation of brass, and the paper lining arrested electrons emitted by aluminium. The brass cylinder had inlet and outlet tubes for the contained gas, and was shielded from external stray radiation by a covering of lead. The

electrode /

electrode inside the chamber consisted of a fine aluminium rod. The chamber was mounted on the secondary electroscope, the gold-leaf system of which was connected with the electrode inside the chamber as shewn in fig 3 (b). The gold leaf, together with this electrode, was initially charged before each exposure to a potential of 240 volts from a dry cell radio battery of the Ever-ready "Winner" type, and the walls of the electroscope and the ionisation chamber were earthed. With this P.D. applied between the electrodes, the ionisation current was saturated.

The ionisation chamber carried in front of the window a lead cylinder, 5.2 cm. long and 2 cm. in diameter, with its axis on that of the ionisation chamber. The secondary electroscope, with the ionisation chamber mounted on it, was placed on a radial, flat wooden arm, which could rotate about a vertical axis at the centre of a circular table (radius 36.3 cm.), graduated in angles at the rim. The electroscope also admitted of a radial movement on this arm, so as to be placed at any desired distance from the centre of the table on which the angle measurements were made. When the suitable distance had been found, the electroscope could be fixed in that position. The readings for the deflections, or rather for the change in the deflections /

deflections of the gold leaf of the electroscope, were taken with a microscope, fitted with a micrometer scale inside it. The micrometer microscopes were of the type manufactured by Messrs Pye & Co., Cambridge. The field of view of the microscope, fixed to the secondary electroscope, was illuminated by means of an electric pea lamp (3.5 volts).

For the primary electroscope, which was situated on a massive tripod stand, a high order sensitiveness was not required; and so it was used without any ionisation chamber filled with SO_2. The ionisation produced in the air of the electroscope itself, which directly received the primary beam, was sufficient for the purpose. The width of the primary beam entering the electroscope could be regulated by means of lead screens with one or more apertures, of suitable size, placed in front of the entrance-window of the electroscope. The primary electroscope, it should be noted here, served only as a standard of comparison. As in the case of the secondary electroscope, the gold leaf of the primary electroscope also was charged, before each exposure, to a potential of 240 volts.

(5). Scatterer.

Five different scatterers were employed in the main investigation; viz. (1) 70 sheets of filter paper (size 21.6 cm. x 21.6 cm., superficial density of each /

each*.0064 gm/cm^2), fixed in a rectangular frame of wood, (2) a thick paraffin-wax sheet (size 20.5 cm. x 19.5 cm., thickness 1.8 cm.), (3) an aluminium sheet (size 20.4 cm. x 20.4 cm., thickness .87 mm.), (4) a carbon slab (size 13 cm. x 14.5 cm., thickness 6 mm.) and (5) a sulphur slab (size 16 cm. x 11.5 cm., thickness 1 mm.). The scatterer could be placed, with faces vertical, in a brass holder capable of rotation about a vertical axis, at the centre of a small wooden disc, graduated in angles, and concentric with the large circular table on which the secondary electroscope, with the ionisation chamber mounted on it, turned. A pointer attached to the holder, indicated on the inner graduated wooden disc the angular position of the scatterer with respect to the axis of the primary beam.

2. Setting and adjustment of the apparatus.

A very important operation, before actually commencing observations, was to set the X-ray tube in the right position. Firstly, the axis of the primary beam passing through the lead outlet tube had to be horizontal. Secondly, as polarisation experiments (to be explained later on under the heading "correction for polarisation") required the tube, or better /

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* Determined and kindly supplied by Mr. J. Reekie.

better, the cathode stream, to be made alternately horizontal and vertical, it was particularly important that this operation did not disturb the horizontality of the axis of the primary beam or the position in space, of the anticathode spot. For this purpose, a fine "X-shaped" paper cross was fixed on the bulb of the X-ray tube, so that the centre of the cross, the centre of the anticathode spot and the centre of the pivot, about which the tube-stand turned, appeared to be in a horizontal line when observed visually, whether the tube was horizontal or vertical. A cross-wire of black thread was also mounted on the tubular aperture for the primary beam, and the height and position of the tube were adjusted, so that, when observed from outside visually, the centre of the cross-wire was in alignment with the above axis of rotation which contained the centre of the anti-cathode spot, for either position (horizontal or vertical) of the tube. This test was further confirmed by placing a fluorescent barium-platinocyanide screen in the path of the primary beam at the edge of the circular table. For both positions of the X-ray tube, the fluorescent patch of light on the screen should remain exactly in the same position.

The horizontal axis of the ionisation chamber was adjusted to the same level as the axis of the primary beam.

The /

The zero line, with respect to which angle Ø was measured, was a standard line and had to be fixed correctly and permanently. This was done by projecting orthogonally the centre of the cross-wire on the circular spectrometer table, and drawing the diameter of the latter through the projection point. The extremity of this diameter, remote from the X-ray tube, was marked zero, and the diameter was the zero line. (When the X-ray tube was in the correct position, the centre of the circular fluorescent patch on the screen, placed at that extremity of the zero-line, was vertically above the zero-line itself.)

The lead cylinder, projecting outward from the ionisation chamber in front of its aluminium window, was also adjusted so as to have its axis passing through the vertical, central axis of the circular base, on which the secondary electroscope turned. A pointer attached to the base of the secondary electroscope and moving over the edge of the graduated circular table, measured the angle Ø, enclosed between the axes of the aforesaid cylinder and the primary beam.

3. Experimental Procedure.

When the X-ray tube had attained a steady condition at the proper voltage, the deflections of the secondary electroscope /

electroscope for $\phi = 90^{\circ}$ and $\phi = \phi$ (say 30°) were alternately noted corresponding to a constant convenient deflection of the primary electroscope. The observations were repeated after turning the radiator through 180° and the mean of the values of $\delta_{\phi}/\delta_{90}$ for the two cases was taken, thus minimising any error due to eccentricity.

Each of the deflections δ_{ϕ} and δ_{90} was observed a number of times, and in practice no reading was accepted unless its deviation from the mean was less than 1%. The smallest deflection of the secondary electroscope was not less than 10 divisions of the micrometer scale. With a little practice, it should be possible to read the deflection correctly to one tenth of a division, so that the maximum error due to reading only should not exceed $^{1}/_{10}$ in 10, i.e.1%

In view of the fact that the radiations used were heterogeneous complex beams and that their different penetrating powers were differently obtained - some by altering the voltage on the X-ray tube without filtering, and some by filtering the radiation at a constant voltage with different thicknesses of aluminium - a convenient way of designating them would perhaps be by their hardness, or what amounts to the same thing, by their average mass absorption coefficient $\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$. This was determined arbitrarily from a 50% absorption in aluminium, according to the relation $I_x = I_o e^{-\bar{\mu}x}$, (where I_x = ionisation produced by the beam /

beam transmitted through a thickness x cm. of an absorbing substance, I_o = ionisation produced by the incident beam, and ρ = density of the absorbing substance), as though the whole radiation was homogeneous.

After having determined the average mass absorption coefficient $(\bar{\frac{\mu}{\rho}})_{Al.}$ we can pass on to attribute to heterogeneous radiation of each hardness, an approximate wavelength, defined as the wavelength of a homogeneous beam of X-ray which has the same $(\bar{\frac{\mu}{\rho}})_{Al}$ as for the heterogeneous beam in question. This average or "equivalent" wavelength λ is obtained by interpolation from a calibration curve, showing the relation between λ and the corresponding $(\bar{\frac{\mu}{\rho}})_{Al.}$ for a monochromatic beam. The data for the calibration curve were obtained from "Spektroskopic der Röntgenstrahlen" by M. Siegbahn, page 231. They are given below in table 2. The method, however, is recognised as quite crude and no high order of accuracy, in relation to wavelength, can be expected from it.

Table 2.

λ(Å.U.)	$(\bar{\frac{\mu}{\rho}})_{Al.}$
.1	.170
.2	.263
.3	.537
.4	1.050
.5	2.040
.6	3.236
.7	5.010
.8	7.310
.9	10.40

It will be explained later that the ratio of the deflections /

deflections, δ_ϕ/δ_{90}, does not give the real ratio of the intensities, I_ϕ/I_{90}. Certain corrections need to be applied. They are enumerated below.

(1) Correction due to stray effect.

(2) Correction due to difference of absorbabilities, of the scattered radiation in different directions. (Compton effect).

(3) Correction due to obliquity of the rays entering the ionisation chamber.

(4) Correction due to polarisation of the primary beam.

No correction was thought necessary for the presence of very soft characteristic radiation in the beams scattered by aluminium and sulphur, for they would, almost completely, be absorbed before reaching the interior of the ionisation chamber.

The following tables 3(a), 3(b), 3(c) and 4 contain the results of observations under different experimental conditions. In these tables the two neighbouring values, one below the other, refer to the two faces of the scattering material.

Table 3(a). $\phi = 150^{\circ}$.

Scatterer - Paraffin wax (1.8cm. thick).

Kilo Volt (peak)	Thickness of Al. filter in mm.	$(\frac{\bar{\mu}}{\rho})$Al.	Mean δ_{150}	Mean δ_{90}	$\frac{\delta_{150}}{\delta_{90}}$	Mean $\frac{\delta_{150}}{\delta_{90}}$ (uncorrected)
30	0	6.55	33.0 31.7	18.25 17.9	1.81 1.77	1.79
80	.54	1.88	32.0 30.9	18.15 17.9	1.76 1.73	1.745
100	.54	1.40	28.7 27.9	16.5 16.45	1.74 1.70	1.72
100	3.16	.70	24.4 23.6	14.0 13.8	1.74 1.71	1.725
100	6.32	.45	22.1 21.2	12.9 12.5	1.71 1.70	1.705
100	9.48	.37	21.95 19.8	12.9 11.8	1.70 1.68	1.69
100	15.80	.32	19.8 18.0	11.9 11.0	1.66 1.64	1.65

Scatterer - Carbon (.6 cm. thick).

80	6.32	.65	28.75 28.2	16.65 16.5	1.73 1.71	1.72

Table 3(a) continued. $\phi = 150°$.

Scatterer - Filter paper (70 sheets).

Kilo Volt (peak)	Thickness of Al. filter in mm.	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$	Mean δ_{150}	Mean δ_{90}	$\frac{\delta_{150}}{\delta_{90}}$	Mean $\frac{\delta_{150}}{\delta_{90}}$ (uncorrected).
30	0	6.55	22.1 24.8	12.25 14.05	1.80 1.77	1.785
80	.54	1.88	27.9 24.45	16.0 13.95	1.75 1.75	1.75
100	.54	1.40	26.0 25.0	15.0 14.65	1.73 1.71	1.72
100	3.16	.70	22.25 21.45	13.1 12.6	1.70 1.70	1.70
100	6.32	.45	19.8 19.8	11.8 11.8	1.68 1.68	1.68
100	9.48	.37	17.6 17.0	10.9 10.5	1.61 1.62	1.615
	Scatterer - Aluminium (.87 mm. thick)					
80	.54	1.88	20.9 20.8	12.55 12.50	1.67 1.67	1.67
	Scatterer - Sulphur (1 mm. thick).					
80	.54	1.88	31.0 26.15	18.90 15.8	1.64 1.66	1.65

Table 3(b). $\phi = 30^{\circ}$.

Scatterer - Paraffin wax (1.8 cm. thick).

Kilo Volt (peak)	Thickness of Al. filter in mm.	$(\frac{\bar{\mu}}{\rho})_{Al}$	Mean δ_{30}	Mean δ_{90}	$\frac{\delta_{30}}{\delta_{90}}$	Mean $\frac{\delta_{30}}{\delta_{90}}$ (uncorrected).
30	0	6.55	30.7 30.25	13.4 13.0	2.29 2.33	2.31
50	0	3.8	32.2 33.1	15.9 16.0	2.03 2.07	2.05
80	.54	1.88	31.75 30.6	16.35 15.85	1.94 1.93	1.935
100	.54	1.40	29.1 29.0	15.55 15.5	1.87 1.87	1.87
100	3.16	.70	25.0 24.05	13.9 13.2	1.80 1.82	1.81
100	6.32	.45	22.25 21.8	12.6 12.2	1.76 1.79	1.775
100	9.48	.37	21.7 20.3	12.4 11.6	1.75 1.75	1.75
100	15.80	.32	19.4 18.7	11.5 11.0	1.69 1.70	1.695
			Scatterer - Carbon (.6 cm. thick).			
80	.54	1.88	35.4 35.5	17.0 17.0	2.08 2.09	2.085

Table 3(b) continued. $\phi = 30^{\circ}$.

Scatterer - Filter paper (70 sheets).

Kilo Volt (peak).	Thickness of Al. filter in mm.	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al.}$	Mean δ_{30}	Mean δ_{90}	$\frac{\delta_{30}}{\delta_{90}}$	Mean $\frac{\delta_{30}}{\delta_{90}}$ (uncorrected).
30	0	6.55	33.5 32.0	10.85 10.2	3.09 3.13	3.11
50	0	3.8	35.75 35.8	14.05 14.0	2.54 2.56	2.55
80	.54	1.88	38.3 29.35	17.1 13.1	2.24 2.24	2.24
100	.54	1.40	32.1 30.2	14.8 14.0	2.17 2.16	2.165
100	3.16	.70	25.4 24.7	12.9 12.35	1.97 2.00	1.985
100	6.32	.45	22.6 21.2	12.1 11.25	1.87 1.88	1.875
100	9.48	.37	19.9 20.0	10.95 11.0	1.82 1.82	1.82
100	15.80	.32	21.9 18.75	12.9 11.0	1.70 1.70	1.70

Table 3(b) continued. $\phi = 30^{\circ}$.

Scatterer - Aluminium (.87 mm. thick).

Kilo Volt (peak).	Thickness of Al. filter in mm.	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$	Mean δ_{30}	Mean δ_{90}	$\frac{\delta_{30}}{\delta_{90}}$	Mean $\frac{\delta_{30}}{\delta_{90}}$ (uncorrected).
50	0	3.80	54.1 51.0	11.1 10.6	4.87 4.81	4.84
80	.54	1.88	46.0 43.2	11.95 11.05	3.85 3.91	3.88
80	3.16	.92	33.0 34.2	10.30 10.8	3.21 3.17	3.19
		Scatterer - Sulphur (1 mm. thick).				
80	.54	1.88	49.4 51.3	11.6 11.75	4.26 4.36	4.31

Table 3(c). $\phi = 20^{\circ}$.

Scatterer - Paraffin wax (1.8 cm. thick).

Kilo Volt (peak)	Thickness of Al. filter in mm.	$(\bar{\mu}/\rho)_{Al.}$	Mean δ_{20}	Mean δ_{90}	$\frac{\delta_{20}}{\delta_{90}}$	Mean $\frac{\delta_{20}}{\delta_{90}}$ (uncorrected)
30	0	6.55	42.15 44.7	12.9 13.65	3.27 3.28	3.275
50	0	3.80	37.05 35.2	13.9 13.05	2.67 2.70	2.685
80	.54	1.88	40.7 35.2	17.1 15.0	2.38 2.35	2.365
80	3.16	.92	30.95 29.95	14.15 13.8	2.19 2.17	2.18
80	6.32	.65	31.0 27.9	14.7 13.05	2.11 2.14	2.125

Scatterer - Filter paper (70 sheets).

30	0	6.55	52.0 55.7	10.6 11.2	4.91 4.97	4.94
50	0	3.80	45.0 47.2	11.4 12.0	3.95 3.93	3.94
80	.54	1.88	43.3 41.3	13.55 12.8	3.20 3.23	3.215
80	3.16	.92	38.6 36.25	13.95 12.95	2.77 2.80	2.785
80	6.32	.65	27.9 30.65	10.90 11.95	2.55 2.57	2.56

Table 4.

Scatterer - Filter paper (70 sheets)

Kilo. volt. (peak) = 80. (Filtered by .54 mm. Al.)

$\left(\frac{\bar{\mu}}{\rho}\right)_{Al} = 1.88$ Equiv. λ = .49 Å.U.

Angle ϕ	Mean δ_ϕ	Mean δ_{90}	δ_ϕ/δ_{90}	Mean δ_ϕ/δ_{90} (uncorrected).
20°	(Vide Table 3(c))			3.215
30°	(Vide Table 3(b))			2.24
40°	38.75	21.7	1.79	1.79
	28.6	16.0	1.79	
60°	32.5	25.65	1.27	1.28
	28.95	22.4	1.29	
90°				1

Correction for stray effects.

Although the major portion of the observed deflection of the secondary electroscope is due to the rays scattered by the radiator alone, yet a small fraction of it owes its origin to the combined effect produced by

(1) radiation scattered by air in the neighbourhood of the scattering substance,

(2) tertiary radiations due to the exposure of directing tubes or edges to either primary or secondary radiations,

(3) possible feeble radiation of the highest frequencies penetrating the lead shields,

(4) a slight natural ionisation in the ionisation chamber and electroscopes.

Effects arising from the above causes we have chosen to call "Stray effects", though the first mentioned, which is the principal contributor to the stray effect, may frequently be regarded as a portion of the scattered radiation and thus not leading to serious error. In spite of this, an attempt has been made to eliminate the effect from the experimental results. The whole stray effect is usually small, though in some cases it amounts to 2 to 3% of the whole. This is by no means negligible, and due account of it has to be taken, especially /

especially where a comparison of the results with theory is sought. Obviously the deflection due to the stray effect must be subtracted from the observed deflection, in order to get the real deflection produced by the radiation scattered by the radiator alone. To make this correction, the deflections of the secondary electroscope were noted, for equal times, with the radiator in position, and with the same removed.

Suppose δ_ϕ and δ_{90} are the observed deflections in a certain time 't', when the ionisation chamber is placed to receive scattered radiations in directions making angles ϕ and 90° respectively with the primary beam, the radiator being in position. Further suppose α and β are two coefficients, such that $\alpha\delta_\phi$ and $\beta\delta_{90}$ represent the corresponding deflections, in the same time 't', with the radiator removed. Then the real deflections due to radiation scattered by the radiator alone are approximately $(\delta_\phi - \alpha\delta_\phi)$ and $(\delta_{90} - \beta\delta_{90})$

Assuming that the intensity I is proportional to the deflection δ, we have,

$$\text{Corrected } (I_\phi/I_{90}) = \frac{\delta_\phi - \alpha\delta_\phi}{\delta_{90} - \beta\delta_{90}} = \frac{\delta_\phi(1-\alpha)}{\delta_{90}(1-\beta)}$$

$$= \frac{\delta_\phi}{\delta_{90}}(1-\alpha+\beta), \quad \alpha \text{ \& } \beta \text{ being small.}$$

The correction factor thus consists of two coefficients α and β, which may be determined by observing the deflections produced /

produced in the same time, with the scatterer in position and without it; and then dividing the latter by the former. (As the primary beams entering the primary electroscope have different constitutions in the two cases, the primary electroscope could no longer be used as a standard of comparison in these experiments.) In practice, δ_{ϕ} and δ_{90° were compared to a definite deflection of the primary electroscope, and the time 't' occupied by this deflection was simultaneously observed. Next, the deflections $\alpha\delta_{\phi}$ and $\beta\delta_{90}$ were determined for the same time 't'. As α and β were small, any small variation in intensity of the primary radiation would be inappreciable in its effect on α and β. Having ascertained α and β, the correction factor $(1 - \alpha + \beta)$ was calculated. The mean values of α and β obtained from 3 readings for each, are shown in Table 5. As this table illustrates, the correction for $\phi = 150^\circ$ was experimentally found to be negligible, and hence $(1 - \alpha + \beta)$ was taken equal to unity there. But for $\phi = 30^\circ$ the correction was not, in all cases, quite negligible; while for $\phi = 20^\circ$, it was greater than for $\phi = 30^\circ$. To reduce the stray effect to a minimum, in the latter case ($\phi = 20^\circ$), it was found necessary to adopt the following device, ie. to place in contact with and on the incidence side of the radiator, a lead screen (20cm. x 20cm. x 1.5 mm.) containing an elliptic aperture just large enough to transmit the whole cone of the primary beam. This was tested by interposing a fluorescent X-ray screen in the path of the beam /

Table 5.

ø	K.V. (peak)	Thickness of Al.filter in mm.	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$	Scatterer	Mean α	Mean β	$(1-\alpha+\beta)$
30°	30	0	6.55	Filter paper (70 sheets)	.050	.032	.982
	50	0	3.80	"	.035	.018	.983
	80	.54	1.88	"	.035	.027	.992
	100	.54	1.40	"	.035	.030	.995
	30	0	6.55	Paraffin (1.8 cm. thick)	.028	.012	.984
	50	0	3.8	"	.015	.006	.991
	80	.54	1.88	"	.017	.013	.996
	100	.54	1.40	"	.012	.010	.998
	80	.54	1.88	Carbon (6 mm. thick)	.023	.014	.991
	50	0	3.8	Aluminium (.87 mm. thick)	.070	.055	.985
20°	30	0	6.55	Filter paper. (70 sheets)	.075	.036	.961
	50	0	3.8	"	.058	.034	.976
	80	.54	1.88	"	.047	.024	.977
	80	3.16	.92	"	.049	.036	.987
	80	6.32	.65	"	.052	.046	.994

Table 5 (continued)

ø	K.V. (peak)	Thickness of Al. filter in mm.	$(\frac{\bar{\mu}}{\rho})_{Al.}$	Scatterer	Mean α	Mean β	$(1 - \alpha + \beta)$
20°	30	0	6.55	Paraffin (1.8 cm. thick)	.047	.015	.968
	50	0	3.80	"	.032	.009	.977
	80	.54	1.88	"	.023	.007	.984
	80	3.16	.92	"	.018	.009	.991
	80	6.32	.65	"	.017	.012	.995
150°	30	0	6.55	Filter paper (70 sheets)	.034	.030	1.00
	80	.54	1.88	"	.027	.026	1.00
	100	.54	1.40	"	.033	.032	1.00
150°	30	0	6.55	Paraffin (1.8 cm thick)	.017	.008	.991
	80	.54	1.88	"	.013	.013	1.00
	100	.54	1.40	"	.012	.0090	1.00

beam transmitted through the elliptic aperture. The residual and reduced stray effect was then corrected for.

It will appear from Table 5 (Pages 42 and 43) that α is greatest for the radiation corresponding to 30 k.v. (peak) and that, with increasing hardness of the incident radiation, it falls, tending to assume a more or less constant value towards the harder region. There is one common and striking feature shewn by the correction factor $(1 - \alpha + \beta)$. This is generally less than unity, since α was found to be greater than β ; but for one and the same scatterer and for the same angle ϕ, it goes on regularly increasing - approaching unity - as the hardness of the radiation increases. Or, in other words, the correction for stray effect goes on diminishing as the incident radiation is made harder, till it becomes negligible.

We have so far been speaking of $(1 - \alpha + \beta)$ as the correction factor. A little reasoning, however, shows that it should generally be equal to $(1 - n\alpha + n'\beta)$ where n and n' are fractions slightly less than unity. For, unless the radiator is very thin, a part of the effective volume of air that scatters, is actually occupied by the radiator; and so far as the air-scattering is concerned, it is not the same with the scatterer present and absent - the air-scattering in the former case is less than in the latter. This makes α and β a little less than the values experimentally determined /

determined. There is another source of error in the determination of α and β ; the stray radiations coming from behind the radiator, ie. from the incidence side, suffer a little absorption while passing through the radiator. This fact makes α and β still smaller, so that the real correction factor is somewhat different from $(1 - \alpha + \beta)$. But n and n′ are not easily determined; and in any case, this correction would be one of second order of magnitude.

In Table 5 no observations have, in general, (with the exception of a few in the case $\phi = 150^{\circ}$) been recorded for α and β , where $(\alpha - \beta)$ was found to be less than .005. The correction factor $(1 - \alpha + \beta)$ in these cases have been assumed unity.

Correction for the difference of absorbabilities of the scattered radiations in different directions.

The deflection δ_ϕ in the gold leaf electroscope is proportional to the ionisation produced in the gas (SO_2) contained in the ionisation chamber, and though a measure of the intensity of the scattered beam, is yet not strictly proportional to it. For, the ionisation is determined not only by the intensity of the incident beam, but also by the absorbability of the radiation. The absorbability of a homogeneous beam is governed by its wavelength λ, - the greater the wavelength, the greater is the absorbability. According to Compton, an incident monochromatic X-radiation suffers, by the process of scattering from light substances, a change in wavelength λ, given by the relation $\Delta\lambda = .0484 \sin^2\frac{\phi}{2}$ Å.U. The absorbability of a scattered beam is thus a function of ϕ, which represents the direction of scattering. A complex heterogeneous beam, made up as it is, of a vast multitude of different homogeneous beams, should according to current theories, exhibit a difference in the absorbabilities after being scattered, varying with the direction of scattering - or, more precisely, as ϕ increases, the corresponding absorbability should increase too, provided of course, there are not other opposing factors /

factors (see later).

It is evident, therefore, that a correction must be applied to "δ_ϕ" for the absorbability, in so far as it depends on ϕ. When this has been done, the corrected δ_ϕ should be proportional to the intensity I_ϕ of the scattered beam. The readiest method that would suggest itself for the calculation of the correction, is that based on Compton's formula of scattering, coupled with the functional relation between the mass absorption coefficient and the wavelength, $\left(\frac{\mu}{\rho}\right) = A + B\lambda^3$, where A and B are constants. In the case of a heterogeneous beam, $\frac{\mu}{\rho}$ and λ might be substituted by their average values. But, the correction thus to be calculated, suffers from the defects that

(1) it presumes that all the incident radiation has been modified by scattering, or that we have a knowledge of the proportion of modified to the unmodified scattered radiation.

(2) an average $\frac{\mu}{\rho}$ for a complex beam is not sufficiently precise; for, unlike the case of a homogeneous beam, the value of $\frac{\mu}{\rho}$ determined experimentally, depends on what fraction of the incident complex radiation has been actually absorbed by the absorbing substance.

(3) the difference in absorbabilities in different directions, e.g. $\phi = 90^\circ$ and $\phi = 30^\circ$, due to the Compton-scattering, may /

may be modified by unequal absorption in the radiator (although the paths traversed in the radiator are adjusted equal in these directions), to an extent depending upon the absorbing power of the radiator. Thus, what difference in absorbabilities is actually present in the radiations entering the ionisation chamber, may not be exactly what was originally brought about by the Compton-scattering.

In addition, much evidence has been obtained in this laboratory, which seems to throw doubt on the validity of Compton's theory as applied to heterogeneous beams.

Under the circumstances, a safer and more direct method of obtaining the correction was preferred to the above method. It is an experimental, rather than a (doubtful) theoretical one. The principle underlying the method is as follows.

Suppose,

I_ϕ and I_{90} be the true intensities of the two beams

x_ϕ and x_{90}, the percentage absorptions in the ionisation chamber;

and Z_ϕ and Z_{90} the ionisations due to the two beams.

Then, assuming the ionisations to be proportional to the energy absorbed, we can put

$$\frac{Z_\phi}{Z_{90}} = \frac{K I_\phi x_\phi}{K I_{90} x_{90}} \quad \text{, where } K = \text{const.}$$

$$= \frac{I_\phi x_\phi}{I_{90} x_{90}}.$$

In order, therefore, to obtain the true value of the ratio I_ϕ / I_{90} /

I_ϕ/I_{90}, we must multiply the observed value Z_ϕ/Z_{90} by a factor x_{90}/x_ϕ

Since, over the range of wavelengths used in these experiments, the absorptions in SO_2 and aluminium are proportional, we can replace x_{90}/x_ϕ by the corresponding quantity for a very thin layer of aluminium of thickness Δt.

For the determination of the quantity x_{90}/x_ϕ, for a thin layer of aluminium, a graphical method may be employed. We can draw two curves, showing the relation between the thickness of intercepting aluminium (in the path of the scattered beams concerned) and (1) ionisation Z_{90}) produced for a certain deflection in the primary electroscope
(2) ionisation Z_ϕ)

From these two curves it is possible to get two quantities $\frac{\Delta Z_{90}}{Z_{90}\Delta t}$ and $\frac{\Delta Z_\phi}{Z_\phi \Delta t}$ independently, at the origin $t = o$; and as the absorption by SO_2 in the ionisation chamber corresponds to a small thickness Δt of aluminium, we may write $x_{90}/x_\phi = \frac{\Delta Z_{90}}{Z_{90}\Delta t} / \frac{\Delta Z_\phi}{Z_\phi \Delta t}$
the L.H.S. quantity is therefore known.

A much more accurate procedure, however, appears to be the one (which was adopted in practice), in which either the numerator or the denominator in the R.H.S. of the above equation is determined directly as already indicated, and the other indirectly from a knowledge of the difference between the /

the two, which can be obtained from a curve showing the relation between z_ϕ/z_{90} $(=\gamma \text{ say})$ and the thickness of intercepting Al. For,

$$\frac{\Delta\gamma}{\gamma\,\Delta t} = \frac{\Delta z_\phi}{z_\phi \Delta t} - \frac{\Delta z_{90}}{z_{90}\Delta t} \quad \ldots \quad \ldots \quad (1)$$

and the quantity $\frac{1}{\gamma}\frac{\Delta\gamma}{\Delta t}$ can be determined from the graph 'y against t'. One of the two R.H.S. quantities being also known, the other is known too, and therewith the value of x_{90}/x_ϕ.

As obviously the degree of accuracy with which x_{90}/x_ϕ can be estimated depends, to a large extent, on the precision with which the two small differences Δz_{90} and $\Delta\gamma$ can be determined from the two curves viz. 'Z_{90} against t' and 'y against t', it is necessary that the different values of Z_{90} and of y should be observed as accurately as possible, corresponding to different thicknesses of absorbing Al. In particular, is this true for y, for which the variation with t is usually small - and consequently it is advisable, in order to make the variation as marked as possible, to extend 't' so as to absorb quite a considerable amount of energy; a 50% reduction in the ionisation was usually chosen.

It need hardly be added, that the greater the deflection δ_{90} to start with (for t = 0), the greater is the accuracy attained in the determination of the different values Z_{90} and y; for, even with a 50% reduction in the ionisation, the deflection will be suffieiently big to admit of accurate observation /

observation. The initial δ_{90} has generally been chosen greater than 20 divisions. This of course, means a very large value for initial δ_{ϕ}, for t= 0, when measured in the same time (ie. for the same deflection in the primary electroscope as in the case of δ_{90}); and indeed, it may some times become so large as to pass beyond the range of observation in the micrometer scale. In such a case the ionisations (say Z'_{ϕ}) for different thicknesses of intercepting Al, were taken corresponding to a smaller - (than in the case of Z_{90}) - but constant deflection in the primary electroscope (without altering the primary aperture). The new ratio Z'_{ϕ}/Z_{90} (= y' say) was plotted against t, instead of y against t, as indicated before. This does not alter the final result; for, the value of $\frac{1}{y'}\frac{\Delta y'}{\Delta t}$ obtained from the curve "y' against t" being obviously equal to $\frac{1}{y}\frac{\Delta y}{\Delta t}$ (since y' is equal to cy, where c is a constant) may be substituted for $\frac{1}{y}\frac{\Delta y}{\Delta t}$ in the equation (1).

In some cases however, where the intensity of the scattered beam was very feeble, the 50% reduction in the ionisation as referred to above could not be adhered to; and the thickness of the intercepting Al. had to be adjusted for a much smaller absorption. Only two points were taken for the graph " y' against t" and they were joined by a straight line.

The results of observations are recorded in Tables 6(a), (b) /

(b), (c) and 7. They are also illustrated by 3 typical graphs, figs 4(a), (b) and (c) in which

(1) γ (or γ') decreases with t

(2) γ (or γ') increases with t

(3) γ (or γ') is independent of t, within errors of experiment.

The correction factors x_{90}/x_{ϕ} given in the following tables, are estimated, in general, to be correct within about 2 per cent.

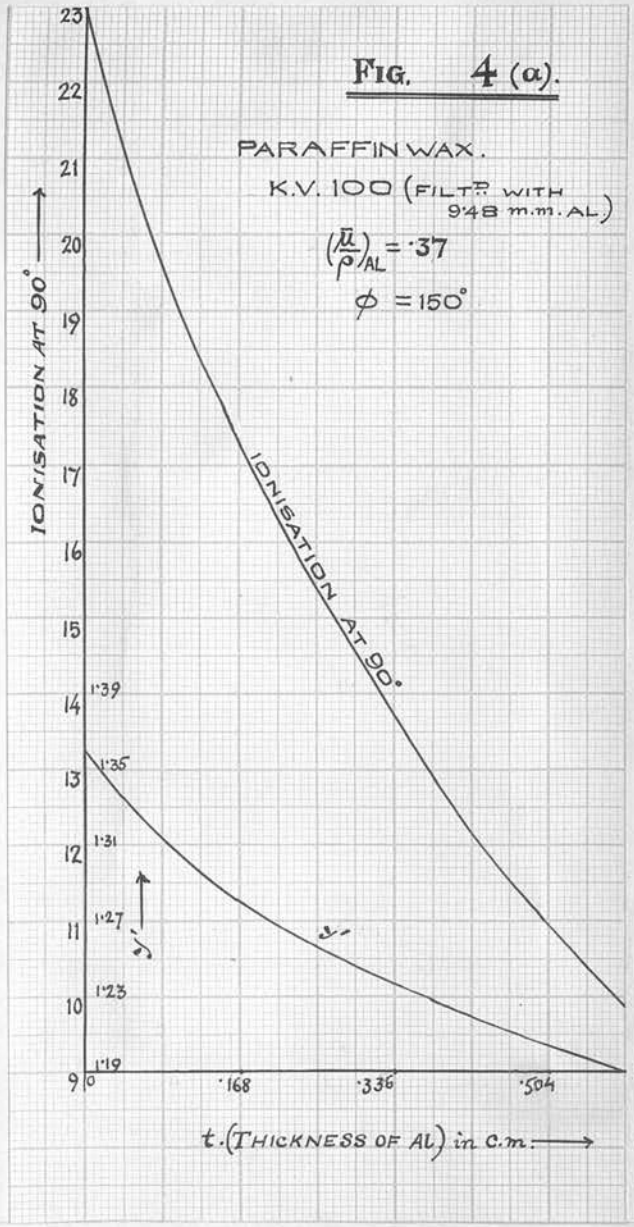


FIG. 4 (a).
PARAFFIN WAX.
K.V. 100 (FILTR. WITH 9·48 m.m. AL)
$(\frac{\bar{\mu}}{\rho})_{AL} = \cdot 37$
$\phi = 150°$
IONISATION AT 90°
IONISATION AT 90°
y'
y'
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
1·39
1·35
1·31
1·27
1·23
1·19
0
·168
·336
·504
t. (THICKNESS OF AL) in c.m.

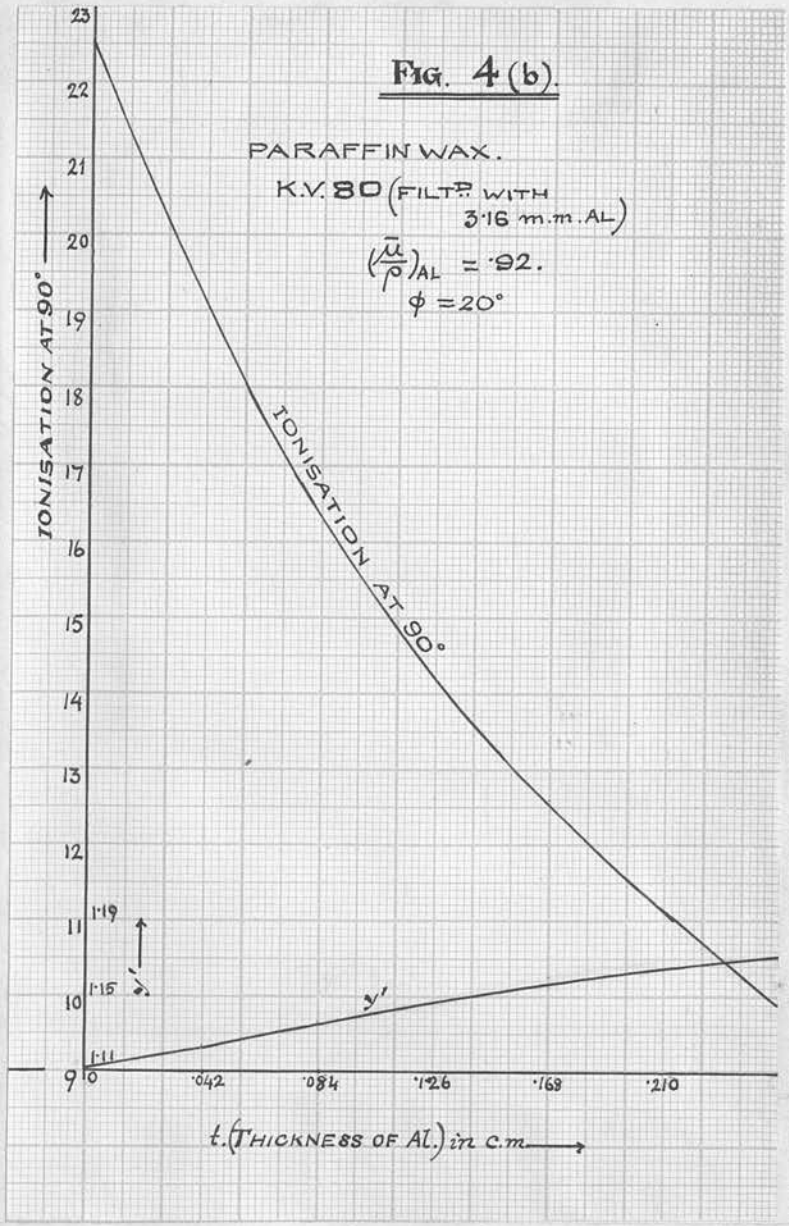


FIG. 4 (b).
PARAFFIN WAX.
K.V. 80 (FILTR. WITH 3·16 m.m. AL)
$(\frac{\bar{\mu}}{\rho})_{AL} = \cdot 92.$
$\phi = 20°$
IONISATION AT 90°
IONISATION AT 90°
y'
y'
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
1·19
1·15
1·11
0
·042
·084
·126
·168
·210
t. (THICKNESS OF AL.) in c.m.

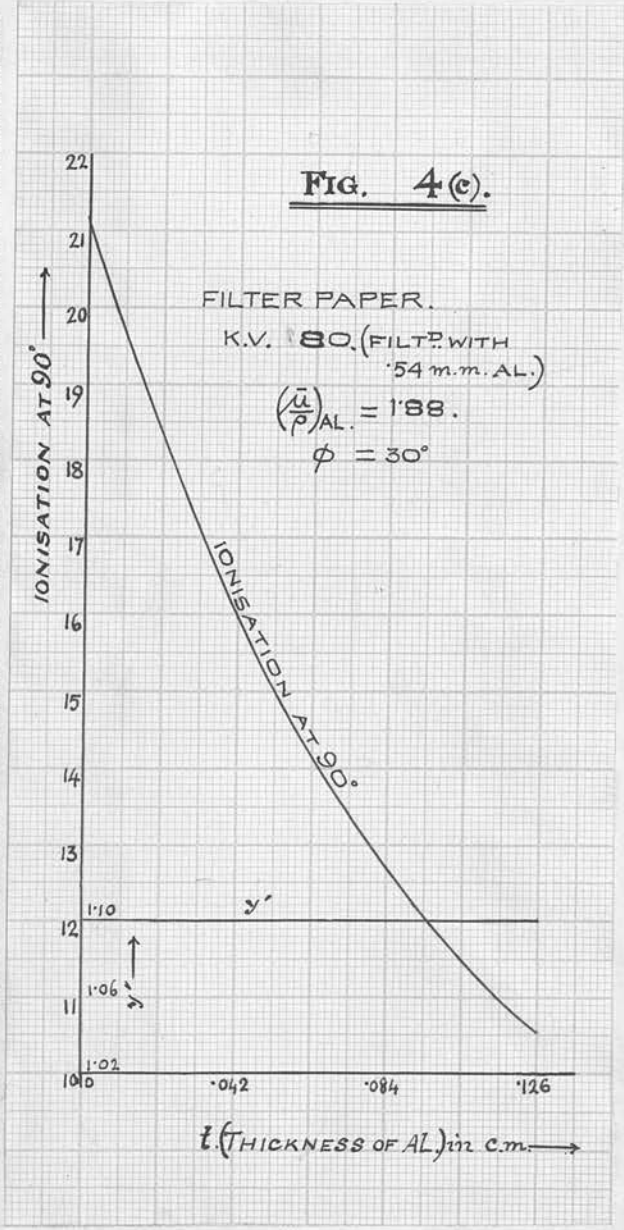


FIG. 4 (c).
FILTER PAPER.
K.V. 80. (FILTR. WITH ·54 m.m. AL.)
$(\frac{\bar{\mu}}{\rho})_{AL.} = 1·88.$
$\phi = 30°$
IONISATION AT 90°
IONISATION AT 90°
y'
y'
22
21
20
19
18
17
16
15
14
13
12
11
10
1·10
1·06
1·02
0
·042
·084
·126
t. (THICKNESS OF AL.) in c.m.

Table 6 (a). $\phi = 150^{\circ}$

Scatterer - Paraffin wax (1.8 cm. thick).

K.V. (peak)	$(\bar{\mu}/\rho)_{Al}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electorscope at 90°	Corrected y (or y′)	x_{90}/x_{ϕ}
30	6.55	0	20.85	1.21	.964
		.01	17.5	1.20	
		.02.	14.85	1.194	
		.04	10.9	1.18	
80 *(filt.54 mm. Al)	1.88	0	25.65	1.16	.91
		.042	19.6	1.13	
		.084	15.65	1.10	
		.126	12.9	1.086	
100 (filt.54 mm.Al)	1.40	0	25.6	1.31	.90
		.042	20.35	1.28	
		.084	16.7	1.26	
		.168	12.0	1.24	
100 (filt. 3.16 mm. Al.)	.70	0	26.15	1.07	.84
		.042	23.25	1.047	
		.084	20.58	1.025	
		.168	16.2	1.015	
		.294	12.1	.988	
100 (filt 6.32 mm. Al.)	.45	0	24.85	1.37	.830
		.084	20.65	1.325	
		.252	14.9	1.275	
		.378	12.0	1.25	
100 (filt 9.48 mm. Al.)	.37	0	23.0	1.361	.818
		.168	17.45	1.281	
		.378	12.93	1.229	
		.588	9.9	1.192	
100 (filt. 15.8 mm. Al.)	.32	0	19.0	1.196	.805
		.126	16.1	1.146	
		.294	13.75	1.084	

*This means that the primary beam at 80 k.v. (peak) has been filtered with .54 mm. Al.

Scatterer - carbon (.6 cm. thick), $\phi = 150^\circ$.

K.V. (peak)	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al.}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90°	Corrected γ (or γ')	x_{90}/x_ϕ
80	.65	0	16.65	1.727	.895
		.042	14.55		
		.084	12.7	1.704	
		.126	11.8	1.67	
		Scatterer - Filter Paper (70 sheets), $\phi = 150^\circ$			
30	6.55	0	19.85	1.207	.97
		.02	13.8	1.192	
		.04	9.8		
80 (filt. .54 mm. Al.)	1.88	0	22.35	1.29	.90
		.042	16.9	1.254	
		.084	13.5	1.23	
		.126	11.0	1.20	
100 (filt. .54 mm. Al.)	1.40	0	21.4	1.285	.88
		.042	16.9	1.25	
		.084	13.75	1.218	
		.168	9.9	1.17	
100 (filt. 3.16 mm. Al.)	.70	0	23.85	1.00	.845
		.084	18.85	.949	
		.168	15.5	.905	
		.294	12.0	.870	
100 (filt. 6.32 mm. Al.)	.45	0	21.6	1.35	.85
		.084	17.85	1.31	
		.210	14.0	1.27	
		.336	11.2	1.22	
100 (filt. 9.48 mm. Al.)	.37	0	16.15	1.62	.87
		.084	13.8	1.59	
		.168	12.05	1.57	
		.252	10.85		
		Scatterer - Aluminium (.87 mm. thick) $\phi = 150^\circ$			
80 (filt. .54 mm. Al.)	1.88	0	22.97	1.110	.947
		.042	17.8		
		.084	14.55	1.079	
		.126	12.47	1.061	

K.V, (peak)	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90°	Corrected γ (or γ')	x_{90}/x_{ϕ}
		Scatterer - Sulphur (1 mm. thick) $\phi = 150^{\circ}$			
80 (filt. .54 mm. Al.)	1.88	0	18.93	1.64	.96
		.042	15.05		
		.084	12.4	1.606	
		.126	10.9	1.593	

Table 6 (b). $\phi = 30^\circ$

Scatterer - Paraffin wax (1.8 cm. thick).

K.V. (peak)	$(\bar{\mu}/\rho)_{Al.}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90°	Corrected y(or y')	x_{90}/x_ϕ
30	6.55	0	20.8	1.54	1
		.01	17.70	1.54	
		.02	15.1	1.56	
		.04	11.75	1.54	
50	3.80	0	28.77	1.030	1.03
		.01	26.0		
		.04	20.2	1.042	
		.07	16.17	1.054	
		.10	13.0	1.075	
80 (filt. .54 mm. Al.)	1.88	0	22.72	1.241	1.03
		.042	17.70	1.254	
		.084	14.10	1.273	
		.126	11.77	1.275	
100 (filt. .54 mm. Al.)	1.40	0	21.57	1.39	1.02
		.042	17.05		
		.084	14.2	1.405	
		.168	10.5	1.40	
		.252	8.0	1.43	
100 (filt. 3.16 mm. Al.)	.70	0	24.15	1.124	1.03
		.084	18.6	1.15	
		.210	13.6	1.15	
		.336	10.37	1.157	
100 (filt. 6.32 mm. Al.)	.45	0	22.0	1.423	1.040
		.126	16.8	1.443	
		.252	13.35	1.449	
		.378	10.90	1.468	
100 (filt. 9.48 mm. Al.)	.37	0	21.4	1.402	1.040
		.126	17.1	1.409	
		.252	14.1	1.425	
		.378	11.9	1.441	

K.V. (peak)	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al.}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90°	Corrected y (or y′)	x_{90}/x_{ϕ}
		Scatterer - Carbon (.6 cm. thick).			$\phi = 30°$
80 (filt. .54 mm. Al.)	1.88	0	17.0	2.082	1.04
		.042	13.43		
		.084	10.97	2.124	
		.126	9.2	2.138	
		Scatterer - Filter Paper (70 sheets)			$\phi = 30°$
30	6.55	0	15.95	1.56	1
		.02	11.50	1.56	
50	3.80	0	21.73	1.52	1
		.04	14.35	1.526	
		.07	11.40	1.513	
80 (filt. .54 mm. Al.)	1.88	0	21.20	1.10	1
		.042	15.95	1.10	
		.084	12.70	1.10	
		.126	10.53	1.09	
100 (filt. .54 mm. Al.)	1.40	0	25.0	1.29	1
		.084	15.85	1.29	
		.210	10.0	1.30	
100 (filt. 3.16 mm. Al.)	.70	0	22.9	1.17	1
		.084	17.65	1.175	
		.168	14.50	1.165	
		.294	11.20	1.16	
100 (filt. 6.32 mm. Al.)	.45	0	21.7	1.493	1.02
		.084	17.8	1.505	
		.210	14.2	1.51	
		.336	11.8	1.51	
100 (filt. 9.48 mm. Al.)	.37	0	15.9	1.824	1.01
		.126	12.75	1.828	
		.210	11.4	1.833	
100 (filt. 15.8 mm. Al.)	.32	0	12.9	1.700	1.046
		.084	11.6	1.707	

K.V. (peak)	$(\frac{\tilde{\mu}}{\rho})_{Al.}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90	Corrected y(or y′)	x_{90}/x_{ϕ}
		Scatterer - Aluminium (.87 mm.thick).			$\phi = 30^{\circ}$
50	3.8	0	13.87	2.42	1
		.01	12.83		
		.04	10.45	2.43	
80 (filt. .54 mm. Al.)	1.88	0	19.5	1.938	.93
		.042	15.6		
		.084	13.1	1.870	
		.126	11.17	1.838	
80 (filt. 3.16 mm. Al.)	.92	0	11.0	1.620	.97
		.042	9.23		
		.084	8.45	1.597	
		Scatterer - Sulphur (1 mm.thick).			$\phi = 30^{\circ}$
80 (filt. .54 mm. Al.)	1.88	0	18.4	2.127	.96
		.042	15.95		
		.084	13.9		
		.168	10.65	2.08	

Table 6 (c)

Scatterer - Paraffin wax (1.8 cm. thick). $\phi = 20^{\circ}$

K.V. (peak)	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al.}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90°	Corrected y(or y')	x_{90}/x_{ϕ}
30	6.55	0	23.73	1.119	1
		.01	19.85		
		.02	17.43	1.110	
		.04	13.55	1.105	
50	3.80	0	23.45	1.317	1.03
		.03	18.00	1.314	
		.05	15.17	1.334	
		.08	12.00	1.345	
80 (filt. .54 mm. Al.)	1.88	0	22.5	1.167	1.035
		.042	17.1		
		.084	13.95	1.188	
		.126	11.50	1.197	
80 (filt. 3.16 mm. Al.)	.92	0	22.53	1.112	1.07
		.042	19.1	1.123	
		.084	16.27	1.136	
		.168	12.57	1.155	
		.252	9.87	1.169	
80 (filt. 6.32 mm. Al.)	.65	0	22.7	1.085	1.080
		.042	20.0		
		.084	17.73	1.108	
		.168	14.4	1.130	
		.210	13.05	1.145	

K.V. (peak)	$(\bar{\mu}/\rho)_{Al.}$	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90°	Corrected y (or y′)	x_{90}/x_{ϕ}
		Scatterer - Filter Paper (70 sheets).			$\phi = 20°$
30	6.55	0	16.0	2.487	1
		.02	11.30	2.478	
50	3.80	0	20.78	1.303	1
		.03	15.0	1.304	
		.055	12.70		
		.07	10.70	1.303	
80 (filt. .54 mm. Al.)	1.88	0	21.77	1.282	1
		.042	16.45		
		.084	12.83	1.29	
		.126	10.50	1.287	
80 (filt. 3.16 mm. Al.)	.92	0	21.05	1.121	1.038
		.042	17.50		
		.084	15.25	1.135	
		.168	11.83	1.154	
80 (filt. 6.32 mm. Al.)	.65	0	15.47	2.559	1.046
		.042	13.45		
		.084	12.0	2.592	

Table 7.

Scatterer - Filter Paper (70 sheets)

K.V. (Peak) = 80. (filt. by .54 mm. Al.). $\left(\frac{\bar{\mu}}{\rho}\right)_{Al} = 1.88.$

Angle ϕ	Thickness of intercepting Al. in cm.	Corrected defl. of secondary electroscope at 90°	Corrected y	x_{90}/x_{ϕ}
20°	(Vide Table 6(c))			1
30°	(Vide Table 6(b))			1
40°	0	21.67	1.787	1
	.084	13.1	1.794	
	.126	10.9	1.784	
60°	0	25.65	1.279	1.02
	.042	19.03		
	.084	15.5	1.293	
	.126	12.83	1.304	
90°				1

A survey of results obtained from experiments on relative absorbabilities (tables 6(a), (b), (c) and 7) will show that,

(1) <u>in the backward direction $\phi = 150^{\circ}$</u>, the scattered radiation is softer than along $\phi = 90^{\circ}$, as predicted by Compton, in every case without exception. This is shown by the curves γ (or γ') plotted against 't', sloping down from left to right. Increasing the hardness of the primary beam, in general, increases the relative absorbability in the direction $\phi = 150^{\circ}$, compared with that at $\phi = 90^{\circ}$. This is also in agreement with the Compton theory; since the smaller the incident wavelength, the greater is the percentage change of wavelength produced by scattering. The observed agreement between the results of experiment and theory is of a qualitative nature only, except that it may be said to be of the right order of magnitude.

(2) <u>in the forward direction</u>, the experimental results may be classified under three distinct types, viz,

(a) that in which the scattered radiation is observed to possess a smaller absorbability than that in the direction $\phi = 90^{\circ}$ - as required by the Compton theory as applied to homogeneous radiation.

(b) that in which the scattered radiation is observed to possess an absorbability equal to that of the radiation /

radiation scattered at right angles to the primary (within experimental error).

(c) that in which the scattered radiation is observed to possess a greater absorbability than the radiation scattered in the direction $\phi = 90^\circ$

The observations (b) and (c) superficially appear contrary to the Compton theory.

Under type (a) come paraffin wax and carbon radiators. The curve y (or y') plotted against 't', (for $\phi = 30^\circ$ and 20° for paraffin wax, and $\phi = 30^\circ$ only for carbon) generally have an ascending course from left to right, showing that the complex scattered radiation in the direction $\phi = 30^\circ$ or 20° is less absorbable than in the direction $\phi = 90^\circ$. Here also the agreement with the theory is only qualitative. In the case of paraffin wax, it may be noted, the corresponding relative absorbabilities, in the direction $\phi = 30^\circ$ and $\phi = 20^\circ$, are approximately equal when the incident radiations are the same (i.e. for $\left(\frac{\bar{\mu}}{\rho}\right)_{Al.} = 6.55$, 3.8 and 1.88). But a considerable increase in the correction factor, in the case of $\phi = 20^\circ$, corresponding to radiations defined by $\left(\frac{\bar{\mu}}{\rho}\right)_{Al.} = .92$ and .65, may be in part ascribed to a relatively higher homogeneity of these radiations, being obtained by heavy filtration at 80 K.V. (peak), instead of at 100 K.V. (peak), as in the case $\phi = 30^\circ$.

Filter paper (70 sheets) is the outstanding instance of type (b). With this substance as scatterer, the curves "y (or y') against t" may be seen to be horizontal, within $\frac{1}{2}$ to /

to 1 per cent., corresponding to all radiations, except the heavily filtered (and consequently more homogeneous) ones, in which latter case, the curves rise up as in type (a). The horizontality indicates equal "absorbability" for the beams scattered at $\phi = 30^{\circ}$ (or 20°) and $\phi = 90^{\circ}$. The evidence of this equality of absorption, in the case of filter paper, in the two directions specified, is in confirmation of the results of Khubchandani (vide his Ph.D. Thesis, 1935) and of Barkla and Khastgir (Phil. Mag. Sept. 1926). In fact it has been observed times without number in this laboratory, in association with the J-phenomenon, that there is not any evidence of the slightest difference in the absorbabilities of the two scattered beams, except by the occurrence of a discontinuity. One point of difference between these results and ours, is that although we confirmed equality of absorption we were unable to reproduce the discontinuity, even by a 50 % absorption of the two beams in question. Possibly, the discontinuities could be discovered by further filtering of the two scattered beams.

In other series of experiments, with filter paper (70 sheets) again, it was observed that, for an incident beam, $(\frac{\bar{\mu}}{\rho})_{Al} = 1.88$, the scattered beam in the direction $\phi = 40^{\circ}$, was also equally absorbable to that at $\phi = 90^{\circ}$, whereas that in the direction $\phi = 60^{\circ}$ was about 2% "less" absorbable than the scattered radiation in the direction $\phi = 90^{\circ}$.

The /

The (c)-type of result was yielded by aluminium (.87 mm. thick) and Sulphur (1 mm. thick). The curve y(or y′) plotted against 't', slopes down from left to right, as in (1), indicating that the scattered radiation for $\phi = 30^{\circ}$ was "softer" than that for $\phi = 90^{\circ}$. This result was at first quite unexpected and disconcerting, as it seemed to contradict established theory. For a long time we were not prepared to accept it. But as repeated experiments conducted with all possible care, persistently pointed to the same conclusion, and as consequences arising from it were of secondary importance (for they would not materially affect the nature of the result) in this investigation, the required correction for the difference of absorbabilities was made in accordance with it.

Another peculiarity noticeable in this connection is that, in the case of both aluminium and sulphur, where this contradictory result occurs, the radiations scattered in the directions $\phi = 150^{\circ}$ and 30° (these directions are equally inclined to the direction $\phi = 90^{\circ}$) are both softer than that along 90° to almost the same extent, and hence equally absorbable with respect to each other. This is seen from the fact that the correction factors " $\varkappa_{90}/\varkappa_{\phi}$ " are respectively .947 and .93 for aluminium; and .96 and .96 for sulphur, the corresponding $\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$ for the incident radiation being the same in each case, viz 1.88. In support of this, it /

it may be cited that Miss Mackenzie (Phil. Mag. Feb.1926) also working with aluminium scatterer, found scattered radiations in the direction $\phi = 60^{\circ}$ and 120° - directions equally inclined to that for which $\phi = 90^{\circ}$ - were, to start with, equally absorbable with respect to each other, and when a difference occurred, it occurred by an abrupt jump or discontinuity. We have also repeated Miss Mackenzie's experiment by absorbing each scattered beam to the extent of 50%, and confirmed her result as to the equality of absorption, but failed to reproduce the J-discontinuity. These experiments of ours related to incident radiations for which $\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$ was 1.88 and 2.33.

A natural suggestion was that the absorption within the material of the scatterer might be responsible for results of type (b) and (c). Absorption, however, could not be the only cause, as was proved by repeating experiment (b) for an incident radiation having $\left(\frac{\bar{\mu}}{\rho}\right)_{Al} = 1.88$, with 24 (instead of 70) sheets of filter paper, and experiment (c) with 4 mm. (instead of 1 mm.) thickness of sulphur and the same radiation. The results were indistinguishable from those for 70 sheets of filter paper and 1 mm. thickness of sulphur respectively, although the corresponding thicknesses, and consequently, absorptions differed so widely.

This apparently capricious nature of the results abtained in course of absorbability experiments, however, does /

does not signify any departure from the accepted theory of scattering. For, this is seen to be associated only with the forward direction; the phenomenon of "Excess Scattering" (to be explained later on) is also connected with the forward direction; hence the anomalous absorbability and excess scattering seem to be intimately related to each other; indeed, one follows directly from the other. Excess scattering superposes on the Compton-effect another effect of opposite sense.

Thus, in the case of paraffin and carbon, excess scattering being small, the superposed effect is small, and therefore the Compton change preponderates. For filter paper, the excess scattering is greater than for paraffin and carbon, and the effect superimposed may actually balance the Compton-effect, as in the case of the smaller angles $\phi = 40^{\circ}$, 30° and 20°. The excess scattering for aluminium and sulphur, on the other hand, is vastly greater than for paper, so that the Compton-effect is relatively weaker compared to the superposed effect, which explains the result of type (c). As the wavelength is made smaller, the Compton change of absorbability increases, whereas the excess scattering diminishes; consequently the Compton effect is the preponderating one. This is borne out by the record of observations in tables 6 (b) and (c).

The above possible explanation is only qualitative. A more precise specification of the superposed effect cannot be discussed here, but will be taken up again, in ~~the~~ a concluding chapter. [See P. 107].

Correction due to Obliquity.

Owing to the finite size of the apertures of the ionisation chamber the rays entering it are not all parallel to the axis of the beam. This necessitates a small correction for the obliquity of the beam. For $\phi = 20^{\circ}$, 30° and 150°, we can assume without much error, that over the small ranges of angle ($2\delta\phi$) considered, i.e. $(\phi - \delta\phi)$ to $(\phi + \delta\phi)$, a decrease in the intensity of the rays on one side of the axis is approximately balanced by an increase in intensity on the other; so, for these angles the correction is practically nil. But in the case of $\phi = 90^{\circ}$, there is an increase of intensity on both sides of the axis. Assuming the intensity to vary as $(1 + \cos^2\phi)$, near 90°, the average intensity in the solid angle subtended by the aperture of the chamber at the centre of the Spectrometer was of the order of magnitude 1.01 I_{90}; since the average value of $\cos^2\phi$ estimated from the dimension of the apparatus, was about .01. The observed value of I_{ϕ}/I_{90} was therefore too low by about 1%. So, the observed value has to be increased by 1% in order to correct it for effects of obliquity of the beam entering the ionisation chamber.

Correction for Polarisation.

The classical expression $I_\phi/I_{90} = (1 + \cos^2\phi)$ and Dirac's expression $I_\phi/I_{90} = \frac{(1 + \cos^2\phi)}{(1 + h\nu/mc^2)^{-3}}(1 + \frac{h\nu}{mc^2} \text{ vers } \phi)^{-3}$, based on wave mechanics, are both deduced under the assumption that the incident beam of X-ray is unpolarised. Practically, however, the heterogeneous primary beam obtained by electronic impact against an anticathode is partially polarised. A correction, therefore, has to be applied to the experimental ratio I_ϕ/I_{90}, for the presence of the polarised component in the incident beam before any comparison can be made between the results of experiments and existing theories. The correction may be calculated as below.

Let us suppose that the partially polarised incident beam is made up of two parts:

(1) an unpolarised part of intensity U, and

(2) a plane polarised part of intensity P. The former can be regarded as equivalent to two rectangular components, each of average intensity $\frac{U}{2}$, one along the direction of the axis of the cathode stream, and the other perpendicular to it, both lying in a plane to which the primary ray is normal. The direction of electric vector, for the polarised part, is along the axis of the cathode stream. Thus, when the X-ray tube is horizontal the cathode stream is horizontal too, and the intensities corresponding to the horizontal and vertical components /

components, are respectively $\frac{U}{2} + P$ and $\frac{U}{2}$. According to the classical theory, when the direction of scattering makes an angle ϕ with the incident ray, the scattered radiation due to the vertical component $(\frac{U}{2})$ and that due to the horizontal component $(\frac{U}{2} + P)$, will have intensities $\frac{KU}{2}$ and $K(\frac{U}{2} + P)\cos^2\phi$, where K is a constant. For, the scattered radiation considered is proceeding in a direction perpendicular to the primary vertical component, whereas it makes an angle $(90^\circ - \phi)$ with the horizontal electric vector in the primary beam.

Thus if x be the ratio I_ϕ/I_{90} for the unpolarised beam and x' that for the observed partially polarised beam, when the cathode stream is horizontal, then

$$x' = \frac{K\frac{U}{2} + K(\frac{U}{2} + P)\cos^2\phi}{K\frac{U}{2}}$$

(to a first approximation, assuming classical theory for polarised part.)

$$= 1 + (1 + \frac{2P}{U})\cos^2\phi$$

and $x = 1 + \cos^2\phi$, since $P = 0$

$$\therefore x' = x + \frac{2P}{U}\cos^2\phi.$$

We note that, the ratio for the partially polarised beam is greater than that for the unpolarised beam by $\frac{2P}{U}\cos^2\phi$; so that, in order to get the latter ratio a quantity $\frac{2P}{U}\cos^2\phi$ has to be subtracted from the experimentally obtained /

obtained value.

From the above it is clear that a method must be devised for the determination of the quantity $\frac{P}{U}$, which means the proportion of the polarised part to the unpolarised. This can be easily done by placing the secondary electroscope at $\phi = 90^{\circ}$, and measuring the ionisation by the deflection of the gold leaf, for two distinct positions of the X-ray tube, viz. (A) with the cathode stream horizontal and (B) with the tube turned through a right angle, so that the cathode stream is vertical. The exact setting and adjustments of the tube for the same have been already described and explained earlier.

If i_A and i_B represent the deflections of the electroscope for the two cases (A) and (B) respectively, then

$$i_A = K'\frac{U}{2}$$

$$\text{and } i_B = K'\left(\frac{U}{2} + P\right), \text{ where } K' = \text{const.}$$

$$\therefore i_B/i_A = 1 + \frac{2P}{U}$$

$$\text{or } \frac{P}{U} = \frac{i_B - i_A}{2\,i_A}.$$

From this, the percentage polarisation $\frac{100\,P}{P+U}$ $\left(=100\,\frac{i_B - i_A}{i_B + i_A}\right)$ also may be calculated. The values of $\frac{P}{U}$ and also the p.C. polarisation, as measured in a short ionisation chamber, have been recorded in the table 8. [Page 77]

The figs 5 (a) and 5 (b) respectively, show the relation between /

FIG. 5 (a).

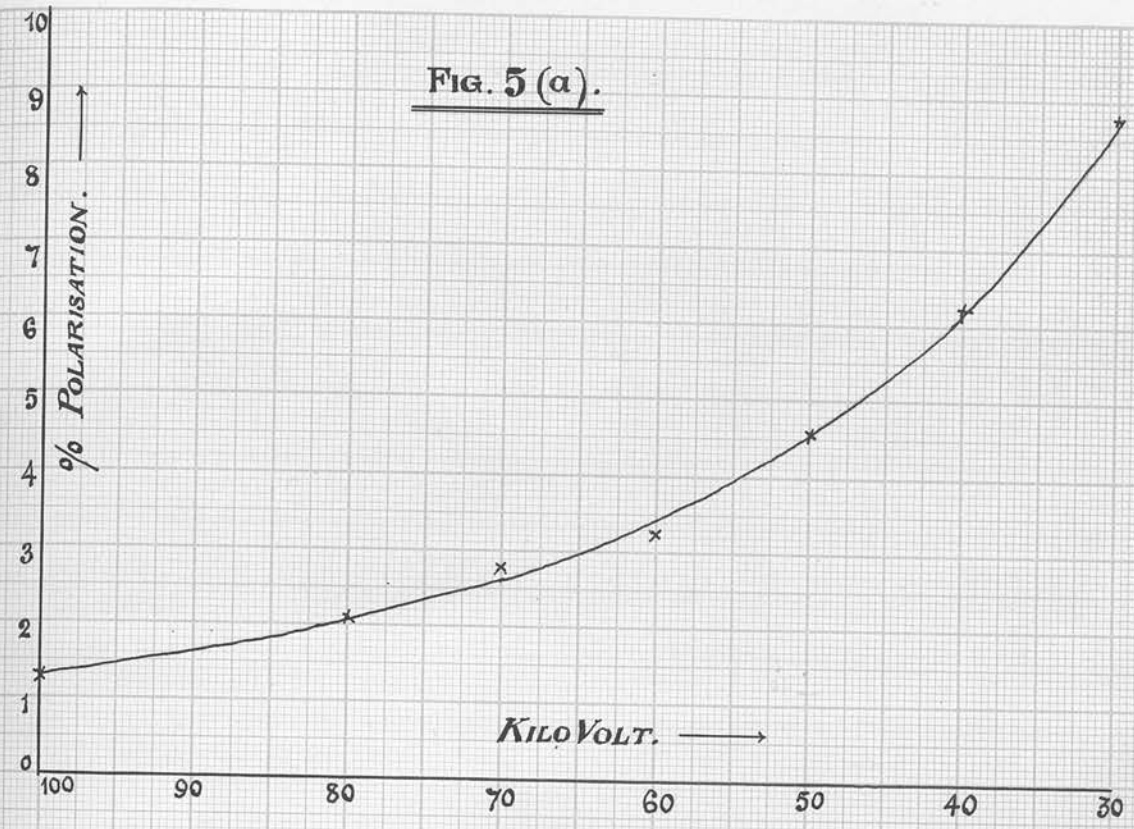
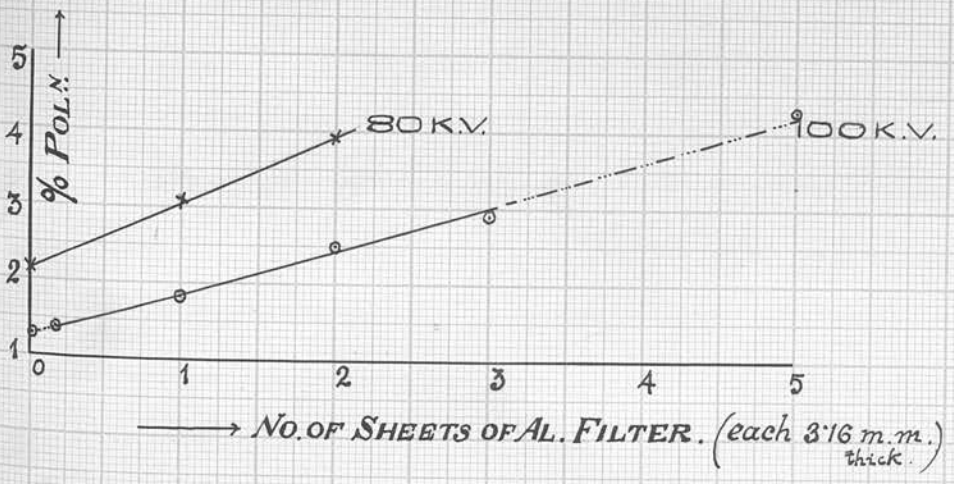

% POLARISATION. ⟶
KILOVOLT. ⟶
0
1
2
3
4
5
6
7
8
9
10
100
90
80
70
60
50
40
30

FIG. 5 (b).


% POL.N. ⟶
80 K.V.
100 K.V.
0
1
2
3
4
5
⟶ NO. OF SHEETS OF AL. FILTER. (each 3·16 m.m. thick.)

between (1) the p.c. polarisation of the unfiltered radiation and the excitation voltage, (2) the p.c. polarisation of the filtered radiations (at the same excitation voltage) and the thicknesses of the filtering aluminium.

From these graphs it may be observed that,

(1) an increase in the excitation voltage decreases the p.c. polarisation of the radiation emitted. Thus in the present case, the effect of raising the excitation voltage from 30 K.V. (peak) to 80 K.V. (peak) brought down the mean p.c. polarisation from 8.8 to 2.1. This is in agreement with the results obtained originally by Barkla (Jahrbuch der Radio-aktivitat und Elektronik V. Band Heft 3, 1908), which showed that the p.c. polarisation varied from about 9 to 2.5 by increasing the P.D. on the tube - and subsequently by Khubchandani (his Ph.D. thesis, 1935), who found that, by raising the voltage from 30 K.V. to 80 K.V. the p.c. of polarisation was lowered from 8.5 to 2.0. This diminution of the amount of polarisation with increasing voltage may be explained as partly due to the fact that at higher voltages a greater proportion of secondary radiation which is unpolarised, is emitted from the anticathode, thus diminishing the proportion of the polarised part.

(2) the rate of change of p.c. polarisation with voltage is greater towards the lower voltage side of the curve than towards the higher voltage side. This means that, so far as /

as changes in the amount of the polarised part in a radiation is concerned, the lower voltage radiation is more sensitive to a variation of the generating voltage.

(3) if the incident radiation at a fixed voltage is hardened more and more by progressive filtration with increasing thickness of the filter (here aluminium), the p.c. of polarisation increases. For instance, in the present case the p.c. of polarisation in a radiation generated at 100 K.V., increased from 1.4 (mean) to 2.9 (mean), when the thickness of the aluminium filter increased from .54 mm. to 9.48 mm; similarly, that in a radiation produced at 80 K.V., went up from 2.1 to 3.95 when the thickness of the aluminium filter was increased from 0 to 6.32 mm.

This is also in qualitative agreement with the results of Barkla (loc. cit.) and Ham (Phys. Rev. 30, 96, 1910), and of Khubchandani (his Ph.D. thesis, 1935). Khubchandani found that, for a radiation at 80 K.V., the proportion of polarisation increased from 2.0 (mean) to 3.03% when the thickness of the aluminium filter was changed from 0 to 5 mm.

The increase in polarisation of the filtered radiation, as we increase the thickness of the filter, may be explained on the supposition that the filter absorbs the softer secondary radiation, leaving a greater proportion of the polarised primary radiation.

(4) /

(4) The relation at constant voltage between the polarisation of the filtered radiation and the thickness of the aluminium filter, appears to be a fairly linear one, at least for a moderate thickness of the filter. This fact has been taken advantage of, in extrapolating the two extreme points on the graph at 100 K.V. (fig 5(b)) as mentioned in table 8. The extrapolation for thickness of 5 sheets of aluminium (each 3.16 mm. thick) at 100 K.V. was necessary on account of the very feeble intensity of the filtered radiation; that for zero thickness on account of the p.c. being too small to be accurately determined by direct experiment.

Since the evaluation of the p.c. polarisation $\left(= 100 \cdot \frac{i_B - i_A}{i_B + i_A}\right)$ involves the measurement of a difference $(i_B - i_A)$ which is usually small, any slight error in the measurement of either or both of them, will produce considerable error in the p.c. polarisation. Hence the results of polarisation experiments, made by a method such as we have adopted here, cannot obviously claim a high percentage order of accuracy. Nevertheless, the correction term containing the quotient $\frac{P}{U}$, being itself small compared to the observed ratio I_ϕ/I_{90}, an error, even of the order of 10%, in the determination of $\frac{P}{U}$ should not vitiate our result of the corrected value of I_ϕ/I_{90} to any appreciable extent.

In view of the above facts, we cannot definitely say whether /

whether the slight difference in the value of $\frac{P}{U}$ observed for the same radiation, with different scatterers - 70 sheets of filter paper, and a paraffin sheet 1.8 cm. thick - is a real one, or is due to experimental error. The difference is so small for the purpose in view, that we are justified in taking the mean of the two values, (corresponding to paper and paraffin), as representing the true value, which we have assumed to hold good in these investigations, for other scatterers as well, e.g. aluminium, carbon and sulphur. This mean value of the p.c. polarisation has been plotted in curve 5 (a), against the corresponding excitation voltage, and in curve 5 (b) against the thickness of aluminium filter. It may not be out of place here, to mention that such a slight difference in the value of $\frac{P}{U}$, as cited above, was also found by Khubchandani for paraffin and carbon radiators. But it could not be determined whether this difference was to be attributed to experimental error, or to any intrinsic variation caused by the material of the radiator.

Again as the intensity of the radiation scattered at $\phi = 90^{\circ}$, by a radiator consisting of 24 sheets of filter paper, was very feeble, it was not possible for him to make direct measurements for $\frac{P}{U}$ in the case of the filter paper scatterer, and he had, therefore, to assume for it what values were obtained for paraffin wax as scatterer. In the present case, however, using 70 sheets of the same filter paper as scatterer, measurements /

measurements could be made directly for the determination of the proportion of polarisation. A comparison between the results for filter paper and paraffin wax recorded in table 8 will justify clearly the above assumption made by Khubchandani.

Polarisation.

Table 8.

(A)

K.V. (Peak)	Thickness of Al. filter.	$\left(\frac{\tilde{\mu}}{\rho}\right)_{Al.}$	Scatterer	Mean i_A	Mean i_B	$\frac{P}{U}$	P.C. Polar-isation.	Mean P.C. polar-isation.
30	0	6.55	Filter Paper.	14.8	17.55	.093	8.5)	8.8
			Paraffin	17.45	20.95	.10	9.1)	
40	0		Paper	20.0	22.7	.068	6.3)	6.25
			Paraffin	26.5	30.0	.066	6.2)	
50	0	3.8	Paper	27.1	29.77	.049	4.7)	4.6
			Paraffin	28.9	31.6	.047	4.5)	
60	0		Paper	28.95	31.0	.035	3.4)	3.25
			Paraffin	28.1	29.9	.032	3.1)	
70	0		Paper	31.35	33.15	.0285	2.80)	2.8
			Paraffin	30.8	32.6	.029	2.80)	
80	0	2.33	Paper Paraffin	31.1	32.45	.022	2.1	2.1
100	0		(Extrapolated from following table (B)).					1.3

(B).

K.V. (Peak)	Thickness of Al. filter.	$(\bar{\mu}/\rho)_{Al}$	Scatterer	Mean i_A	Mean i_B	$\frac{P}{U}$	P.C. Polarisation.	Mean P.C. Polarisation.
100	.54 mm.	1.4	Paper	29.8	30.60	.014	1.3)	1.4
			Paraffin	33.0	34.05	.016	1.5)	
"	.316 cm.	.70	Paper	26.0	27.0	.019	1.9)	1.8
			Paraffin	28.9	29.9	.017	1.7)	
"	.632 cm.	.45	Paper	14.5	15.25	.026	2.5)	2.5
			Paraffin	26.1	27.45	.026	2.5)	
"	.948 cm.	.37	Paper	11.55	12.2	.029	2.8)	2.9
			Paraffin	24.3	25.85	.032	3.0)	
"	1.580 cm.	.32	(Extrapolated from above).			.045		4.3
(c)								
80	.54 mm.	1.88	Paper	31.4	32.9	.024	2.3)	2.25
			Paraffin	35.6	37.25	.023	2.2)	
80	.316 cm.	.92	Paper	24.25	25.75	.031	3.0)	3.1
			Paraffin	35.0	37.3	.033	3.2)	
80	.632 cm.	.65	Paper	14.8	16.0	.041	3.9)	3.95
			Paraffin	24.0	26.0	.042	4.0)	

Results of Observation.

The following tables 9 (a), (b) and (c) contain the results of observation - both uncorrected and corrected - for different radiators viz, (1) Paraffin wax (1.8 cm. thick), (2) Carbon (.6 cm. thick), (3) Filter paper (70 sheets, superficial density of each, .0064 gm/cm.^{2} approx.), (4) Aluminium (.87 mm. thick), (5) Sulphur (1 mm. thick), corresponding to different angles ($\phi = 150^\circ$, 30° and 20°) and to radiations of different average frequencies. For the sake of comparison, the corresponding values calculated on the formula proposed by Dirac are also annexed.

The results of observation (corrected) are also illustrated by curves where I_ϕ/I_{90} has been plotted against

(1) the equivalent wavelength as defined on Page 30, for $\phi = 150^\circ$ (fig 6), $\phi = 20^\circ$ and 30° (fig 7 (b)) P. 90(a)

(2) the cube of the equivalent wavelength, for $\phi = 20^\circ$ and 30° (fig 7 (c)). P. 92 (a).

Table 9 (a).

Table 9 (a)

Angle $\phi = 150^{\circ}$.

Kilo Volt (peak)	$(\bar{\mu}/\rho)_{Al}$	Equiv.* λ A.U.	Scatterer	Uncorrected I_{150}/I_{90}	Corrected I_{150}/I_{90}	Dirac's Theory.
30	6.55	.77	Paraffin	1.79	1.58	1.62
			Filter paper	1.785	1.61	
80 (filt. .54 mm. Al.)	1.88	.49	Paraffin	1.745	1.565	1.55
			Filter paper	1.75	1.56	
			Aluminium	1.67	1.56	
			Sulphur	1.65	1.57	
100 (filt.. .54 mm. Al.)	1.40	.44	Paraffin	1.72	1.54	1.53
			Filter paper	1.72	1.51	
100 (filt. 3.16 mm. Al.)	.70	.34	Paraffin	1.725	1.44	1.47
			Filter paper	1.70	1.43	
100 (filt. 6.32 mm. Al.)	.45	.275	Paraffin	1.705	1.39	1.43
			Filter paper	1.68	1.40	
100 (filt. 9.48 mm. Al.)	.37	.25	Paraffin	1.69	1.35	1.40
			Filter paper	1.615	1.37	
100 (filt. 15.8 mm. Al.)	.32	.225	Paraffin	1.65	1.28	1.36
			Filter paper			
80 (filt. 6.32 mm. Al.)	.65	.33	Carbon	1.72	1.49	1.47

*Determined from average $\hat{}^{mass}$ absorption coefficients, as described

on page 30

Table 9 (b).

Angle $\phi = 30^{\circ}$.

Kilo Volt (peak)	$(\frac{\tilde{\mu}}{\rho})_{Al.}$	Equiv. λ A.U.	Scatterer	Uncorrected I_{30}/I_{90}	Corrected I_{30}/I_{90}	Dirac's Theory.	Empirical Formula. $I_{\phi}/I_{90} = a+b\lambda^3$
30	6.55	.77	Paraffin	2.31	2.16	1.90	2.16
			Filter paper	3.11	2.94		2.94
50	3.80	.635	Paraffin	2.05	2.04	1.93	2.04
			Filter paper	2.55	2.46		2.50
			Aluminium	4.84	4.74		4.72
80 (filt. .54 mm. Al.)	1.88	.49	Paraffin	1.935	1.97	1.98	1.96
			Carbon	2.085	2.13		
			Filter paper	2.24	2.21		2.21
			Aluminium	3.88	3.61		3.63
			Sulphur	4.31	4.14		
100 (filt. .54 mm. Al.)	1.40	.44	Paraffin	1.87	1.90	2.01	1.94
			Filter paper	2.165	2.16		2.12
100 (filt. 3.16 mm. Al.)	.70	.34	Paraffin	1.81	1.85	2.08	
			Filter paper	1.985	1.98		2.03
100 (filt. 6.32 mm. Al.)	.45	.275	Paraffin	1.775	1.82	2.17	
			Filter paper	1.875	1.89		
100 (filt. 9.48 mm. Al.)	.37	.25	Paraffin	1.75	1.79	2.22	
			Filter paper	1.82	1.81		
100 (filt. 15.8 mm. Al.)	.32	.225	Paraffin	1.695	?	2.28	
			Filter paper	1.70	1.72		
80 (filt. 3.16 mm. Al.)	.92	.38	Aluminium	3.19	3.08	2.05	3.08

Table 9 (c).

Angle $\phi = 20^{\circ}$.

Kilo Volt (peak)	$\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$	Equiv. λ Å.U.	Scatterer	Uncorrected I_{20}/I_{90}	Corrected I_{20}/I_{90}	Dirac's Theory	Empirical Formula. $I_{\phi}/I_{90} = a+b\lambda^3$.
30	6.55	.77	Paraffin	3.275	3.04	2.05	2.99
			Filter paper	4.94	4.65		4.70
50	3.80	.635	Paraffin	2.685	2.64	2.09	2.63
			Filter paper	3.94	3.80		3.76
80 (filt. .54 mm. Al.)	1.88	.49	Paraffin	2.365	2.40	2.14	2.39
			Filter paper	3.215	3.13		3.14
80 (filt. 3.16 mm. Al.)	.92	.38	Paraffin	2.18	2.28	2.24	2.27
			Filter paper	2.785	2.83		2.83
80 (filt. 6.32 mm. Al.)	.65	.33	Paraffin	2.125	2.23	2.29	
			Filter paper	2.56	2.61		

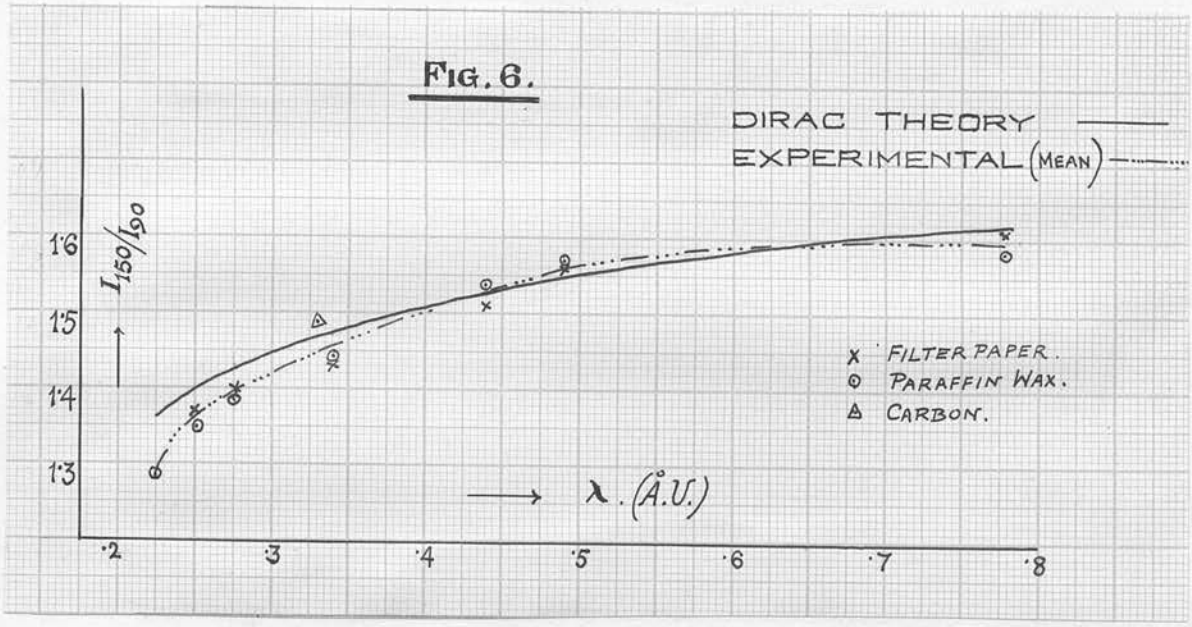
Discussion.

A discussion of the experimental results may conveniently be divided into two sections, the one relating to the backward direction $\phi = 150^{\circ}$; the other to the forward direction $\phi = 30^{\circ}$ and 20°.

Section I. $\phi = 150^{\circ}$.

According to the simple classical theory, the ratio I_{ϕ}/I_{90} should be independent of the nature of the radiator. In addition, the ratio I_{150}/I_{90}, on the above theory has a value 1.75 for radiations of all wave lengths, whereas Dirac's value diminishes continually as the wavelength is shortened - the simple classical value being realised in Dirac's theory only in the limiting case $\lambda = \infty$

A survey of the results of experiment in table 9 (a) shows that for any one radiation the corrected values of the ratio I_{150}/I_{90} for the different scatterers are nearly equal and the mean of these values agrees in general, remarkably well with Dirac's theory, not only in sense, but in magnitude also. This is illustrated by fig. 6.



In the case of aluminium and sulphur, the two elementary radiators, only one radiation - the most intense one ($\lambda = .49$ Å.U.) - had been scattered as the scattered rays in other cases were of extremely feeble intensity.

Dirac's theoretical values have, of course, been calculated here on the assumption that the incident complex beam is analogous to a homogeneous one of a wavelength defined by what has been called "equivalent wavelength", and that it is completely modified by the process of scattering. Indeed, for scattering substances consisting of light atoms such as carbon, paraffin wax, filter paper, etc., the above assumption of a more or less complete modification, is not far from the truth, particularly when the primary beam is of a short wavelength. The real theoretical values I_{150}/I_{90}, however /

however, may be slightly greater for long wavelengths, by an amount depending on the actual percentage of unmodified rays present - an amount of which we really do not possess an accurate knowledge. Considering the possible errors in estimating the various factors involved, the agreement may, on the whole, be looked upon as distinctly satisfactory.

It may be recalled that Backhurst, scattering monochromatic rays of wavelengths .395 and .31 Å.U. from beryllium and other substances, found in the backward direction, as far as 150°, an agreement within about 3% with Dirac's Theory, in every case.

In the investigation described in this paper, the experimental value of the relative intensity I_{150}/I_{90}, for paraffin wax, corresponding to the shortest wavelength $\lambda = .225$ Å.U. falls short of the theoretical by about 6%. Such a discrepancy is more than can be attributed to experimental error alone; it probably means something more. Moreover, a peculiarity noticed in common with Backhurst's results is that the discrepancy between experimental and theoretical values is generally in one direction, the experimental falling below the theoretical value. This is more pronounced in the region of short wavelengths.

By a reference to table 1, it will be seen that, so far as carbon and paraffin radiators were concerned, the results /

results of Khubchandani also fitted in well with Dirac's theory for unfiltered radiations, but quite a big divergence - (experimental value greater than the theoretical by about 9%) - was manifested for the filtered radiation at 80 K.V. Such a divergence we could not confirm for paraffin wax or carbon radiators. Although the incident radiation was progressively filtered more and more in the case of paraffin wax, yet a fairly close agreement between Dirac's theory and the experimental results persisted over quite a big range of wavelengths. In the case of carbon also, using a heavily filtered primary radiation at 80 K.V., we could not detect any appreciable departure from Dirac's theoretical value, even though this radiation was more filtered and consequently harder than the hardest used by Khubchandani, as will be evident from the value of $(\frac{\bar{\mu}}{\rho})_{AL.}$ which in the present case was .65, against .71 in the experiment of Khubchandani.

Then again, in the case of filter paper, for the softest radiation at 30 K.V., his result turned out very near that predicted by the theory, while for harder rays a remarkably peculiar behaviour was observed, viz., that the ratio began to increase as the incident radiation was made more penetrating and eventually reached the classical value 1.75 for $(\frac{\bar{\mu}}{\rho})_{AL.} = .71$. Such a peculiarity for filter paper also we were unable to reproduce and confirm even over such a /

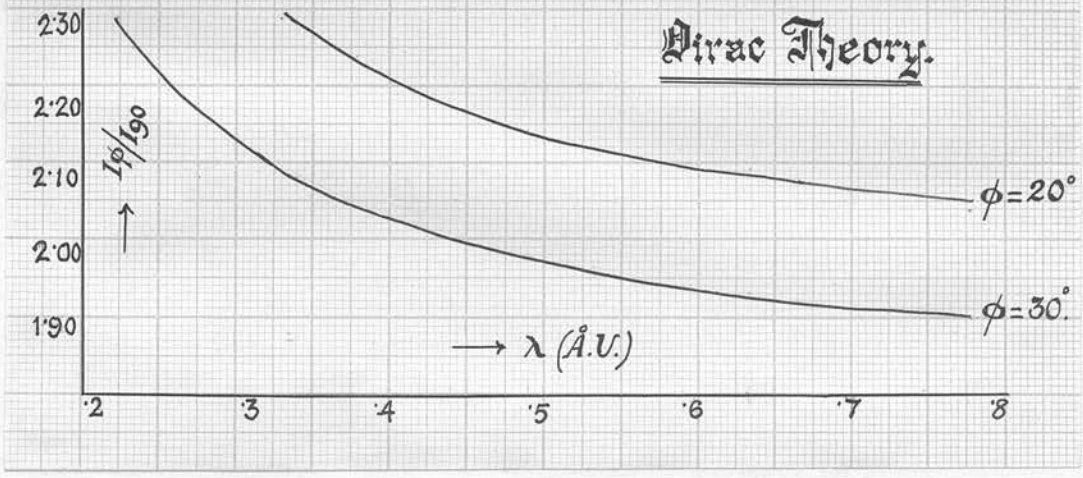
a big range of $\left(\frac{\bar{\mu}}{\rho}\right)_{Al}$ as 1.88 to .37. Moreover, if the fundamental processes occurring in the phenomenon of scattering be the same for all substances, it is difficult to understand why filter paper should behave differently from carbon or paraffin wax. It may further be noticed from table 1 that 5 out of 9 values of the ratio obtained by Khubchandani agree with theory, while the remaining four do not. We have, on the contrary, found a quantitative agreement uniformly good in almost all the cases investigated. Khubchandani laid stress on the result he obtained, that in the backward direction (for $\phi = 150^{\circ}$ at any rate) the ratio was almost a constant for paraffin wax and carbon for all radiations. In the present case, however, we observed a gradual fall in the ratio with the increase in the hardness of the incident rays, in accordance with the theory of Dirac. Such a fall is shown, in our investigation, even by the uncorrected values, in the case of filter paper and paraffin wax.

That in the backward direction the ratio I_{ϕ}/I_{90} suffers a diminution with the increase in the penetrating power of the primary incident rays was, as we have seen before, also observed as early as 1908 by Barkla (Phil. Mag. Feb. 1908). He found that the value of I_{170}/I_{90} came down from about 2 to 1.5 by increasing the hardness of the incident radiation.

Section II. (a). $\phi = 30^\circ$.

The simple classical theoretical value of I_{30}/I_{90} is 1.75, and is a constant for all scattering substances and for radiations of all wave-lengths. The quantum theory of Dirac, however, while retaining the scattering function unaffected by the nature of the scattering substance (under certain limitations), makes it dependent on the wave length of the incident radiation - assumed homogeneous. Dirac's relation between the relative scattered intensity and the incident wavelength has already been shown on page 8 from which it may be noted that I_ϕ/I_{90} - for $\phi < 90^\circ$ - decreases as the wavelength of the incident radiation is increased, and reaches the limiting classical value for $\lambda = \infty$. This is illustrated graphically in fig 7(a).

Fig. 7 (a).



The experimental results obtained by us in course of investigation with different scattering substances, different radiations and different angles are completely at variance with the above theories. In the first place, we have found that, for the same incident radiation, and corresponding to the same angle of scattering, the ratio I_{ϕ}/I_{90} depends, in a large measure, on the nature or physical constitution of the scattering substance. Employing the same incident radiation of equivalent wavelength .49 Å.U. (exciting voltage .80 K.V. peak, filtered with .54 mm. of Al.) we obtained for different scatterers the values of I_{30}/I_{90} given in table 10.

Table 10.

Scattering substance.	Atomic number.	I_{30}/I_{90}
Paraffin wax (1.8 cm. thick)	[<6]*	1.97
Carbon (.6 cm. thick)	6	2.13
Filter paper (70 sheets)	[>6]	2.21
Aluminium (.87 mm. thick)	13	3.61
Sulphur (1 mm. thick)	16	4.14

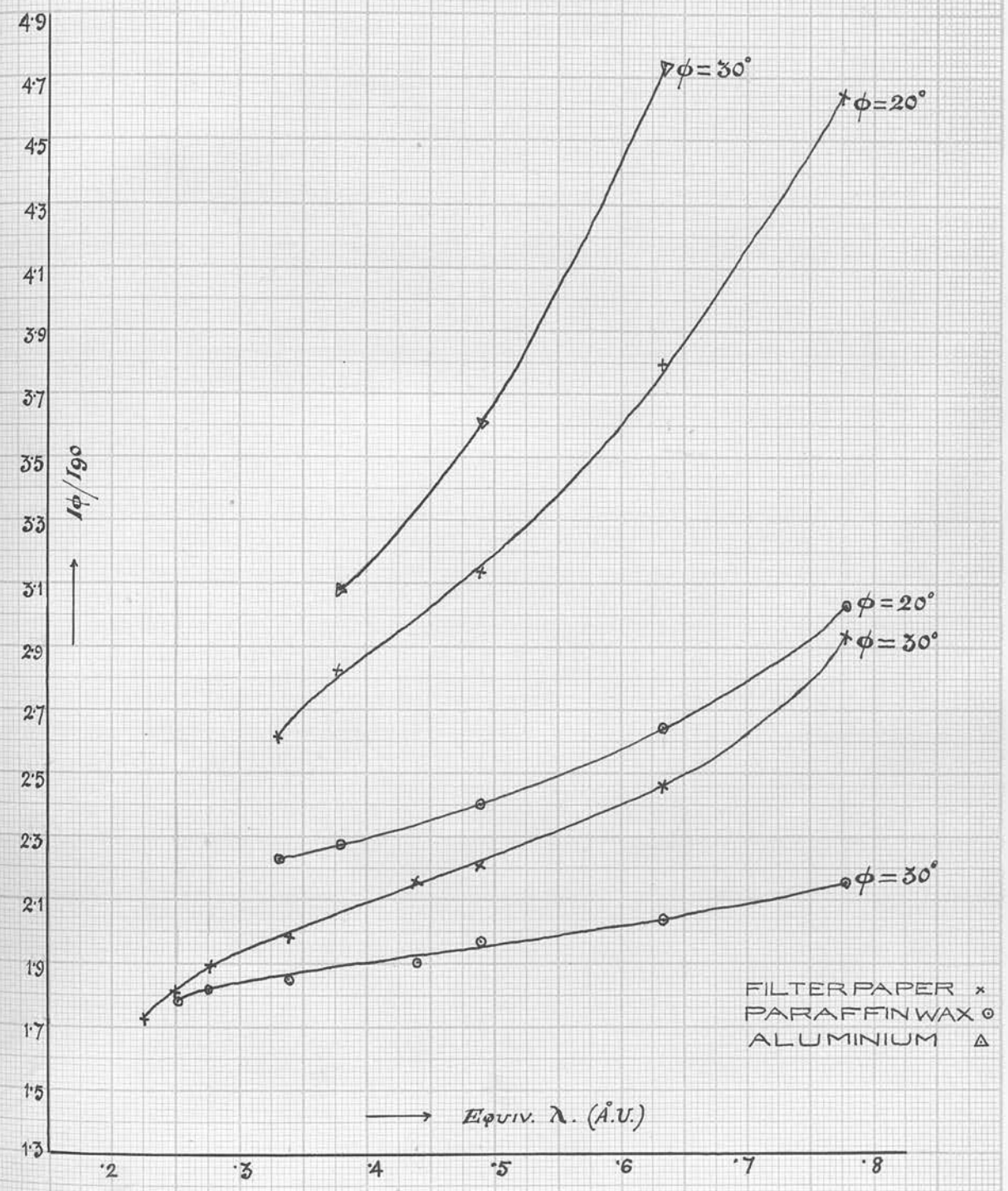
Thus /

*For paraffin wax and filter paper, which are not elementary substances, we can only suggest an average atomic number calculated from their chemical composition.

Thus the relative scattering - certainly as far as these experiments go - increases with the atomic number of the scattering element. This dependence on the nature of the scattering substance, shown by radiations of other wavelengths also, is vividly brought in fig. 7 (b)., where the same order of succession as above has been maintained, beginning from a wavelength of .77 Å.U., down to .225 A.U.

Secondly, the value of I_{30}/I_{90} was found always to be in excess of the simple classical value. This excess was also found to be greater, the greater the wavelength of the incident radiation. All curves illustrated in figs. 7 (b) and (c) slope down from right to left, showing that with the progressive hardening of the incident rays the ratio I_{30}/I_{90} undergoes a continual diminution for each scatterer, approaching the classical limit 1.75 and possibly eventually reaching it. That in the case of filter paper and paraffin wax, this classical limit has been more or less realised, within experimental error, at the hardest end of the curves, is quite apparent from table 9 (b) and figs. 7 (b) and (c). But in the case of an aluminium scatterer, the extremely feeble intensity of the scattered rays, rendered it impossible to make measurements corresponding to wavelengths shorter or longer than .38 and .635 Ångström units respectively. Nevertheless, the curve has manifestly a tendency to slope down /

FIG. 7(b).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φ=20°
φ=20°
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φ=30°
FILTER PAPER ×
PARAFFIN WAX ⊙
ALUMINIUM △
Εφνιν. λ. (Å.U.)
·2
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·4
·5
·6
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down, on the shorter wavelength side, so as to approach the simple classical limit. Such a result, obviously, is not in conformity with Dirac's theory, but is in the opposite sense.

(b) $\phi = 20^{\circ}$.

The simple classical value I_{20}/I_{90} is 1.88, and neglecting for the moment the effects of superposition, is the same for all scatterers. Actual experiments, however, yielded results which have features similar to those described in connection with $\phi = 30^{\circ}$, showing thereby, that these features are not peculiar to any particular angle ϕ, but are probably true, in general, for all the small scattering angles. The difference is one of magnitude and not of kind. I_{20}/I_{90} was found to be very much greater than the corresponding value I_{30}/I_{90}, but the relative positions of different scatterers was unchanged - for filter paper and paraffin wax at least.

In the experiments at $\phi = 20^{\circ}$, the maximum potential applied was 80 K.V., as it became evident that the X-ray tube could not safely stand higher potentials. The larger frequency radiations were obtained by filtering the primary rays excited at 80 K.V., with the result that the intensity of /

of the incident radiation became much feebler. Consequently no observations could be taken beyond a 'wavelength' .33 Å.U. towards the short wavelength side.

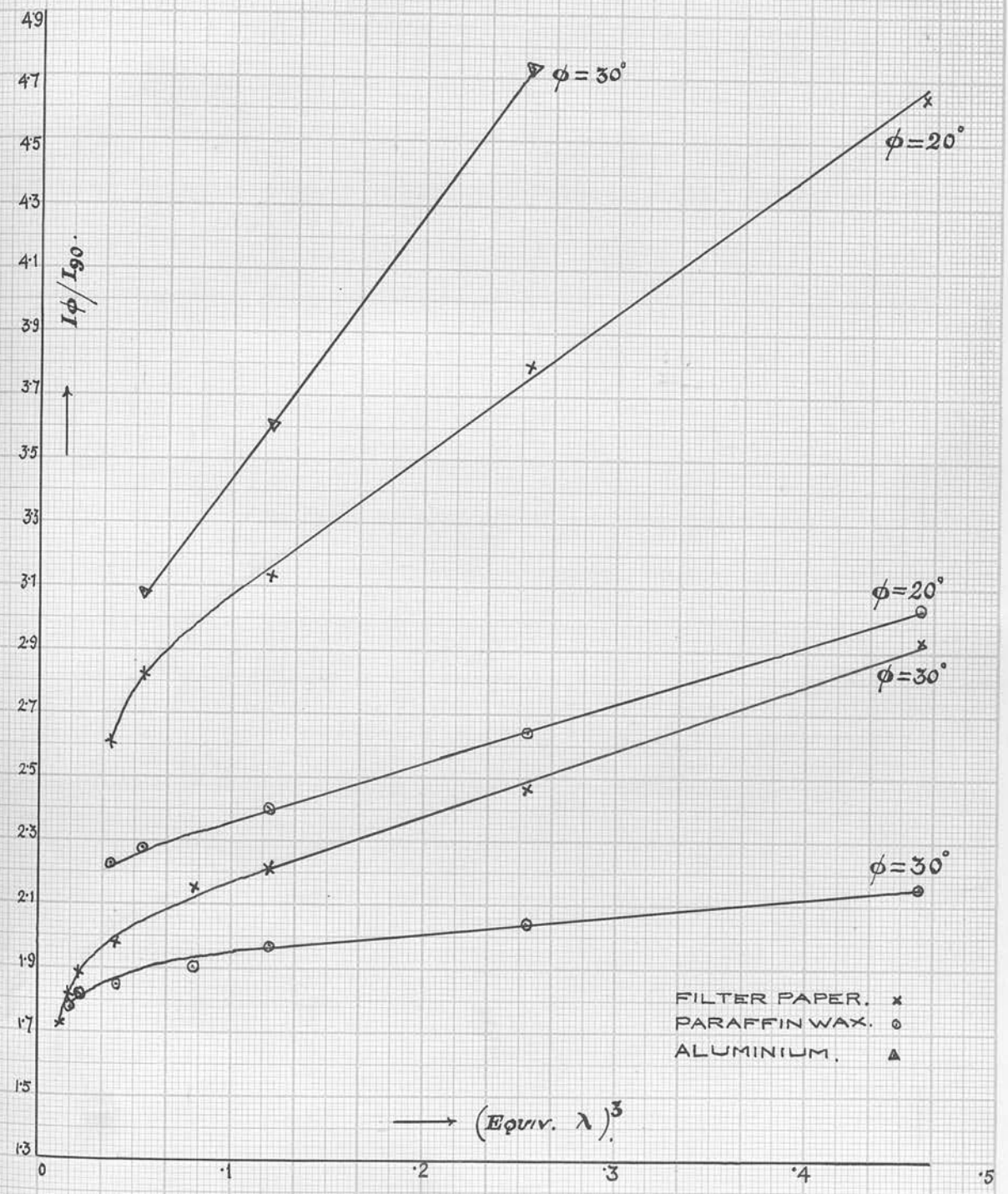
The curves presented in fig. 7 (b) where I_{ϕ}/I_{90} is plotted against the equivalent wavelength (as defined before) appear to resemble one another whether ϕ is 30° or 20°. When I_{ϕ}/I_{90} is plotted against the cube of the equivalent wavelength as in fig. 7 (c), the curves are fairly linear throughout quite an extensive region of longer wavelengths, where the relation between I_{ϕ}/I_{90} and the equivalent λ, can be expressed by an equation of the form $I_{\phi}/I_{90} = a + b\lambda^3$ where 'a' and 'b' are constants, each being a function of (1) the angle ϕ, (2) the nature of the scatterer, and possibly of (3) the tube emitting the radiation. The values of 'a' and 'b' are indicated below.

Table 11.

Scatterer	ϕ	a	b
Paraffin wax (1.8 cm. thick)	30°	1.89	.59
	20°	2.18	1.76
Filter paper (70 sheets)	30°	1.95	2.15
	20°	2.59	4.6
Aluminium (.87 mm. thick)	30°	2.65	8.16

It /

FIG. 7(c).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(Equiv. λ)3
φ = 30°
φ = 20°
φ = 20°
φ = 30°
φ = 30°
FILTER PAPER. ×
PARAFFIN WAX. ⊙
ALUMINIUM. △

It will be observed from this table that 'a' and 'b' are both greater for $\phi = 20^{\circ}$ than $\phi = 30^{\circ}$, for the same scatterer; and that for the same angle ϕ, they are in the same order of succession for different scatterers as in table 10 , being greatest in the case of aluminium and smallest in the case of paraffin wax. The constant 'b' measuring the 'slope' of the straight lines, shows how rapidly the ratio I_{ϕ}/I_{90} changes with 'wavelength' in the region of longer wavelengths extending in the case of paper and paraffin, from $\lambda^3 = .46$ to $\lambda^3 = .07$ (A.U.)^3 approximately, and in the case of aluminium certainly from $\lambda^3 = .256$ to $\lambda^3 = .055$ (A.U.)^3. In fact, 'b' is a measure of the sensitivity of the ratio I_{ϕ}/I_{90} to wavelength change for the substance concerned. Thus for aluminium 'b' is 8.16, whereas for paraffin wax it is .59 only, showing that aluminium is enormously more sensitive to changes of wavelength than paraffin.

The value of the ratio I_{ϕ}/I_{90} calculated from the empirical formula $I_{\phi}/I_{90} = a + b\lambda^3$, have been recorded in the last column of tables 9 (b) and (c) against the corresponding wavelengths. A comparison between this and the experimental value, shows that there is an agreement between them within about 1%.

The rectilinear course of the curves is discontinued near /

near $\lambda^3 = .06$, and falls more quickly within a very small region of wavelengths, at the short wavelength side, till the classical limit is reached. This is seen very prominently in the case of filter paper and less markedly in the case of paraffin wax in fig. 7 (c), where $\phi = 30^\circ$. For paraffin, of course, the curvature is less clear on account of the very low sensitiveness which it has for changed wavelengths. In the case of aluminium, unfortunately, the further course of the curve beyond $\lambda = .38$ Å.U., could not be explored owing to the extremely feeble intensity of the scattered rays, and hence the existence of the bend which should presumably be very pronounced, could not be demonstrated there. In the case of filter paper, for $\phi = 20^\circ$, on account of the feeble intensity of the scattered radiation, we had to stop just where the curvature was beginning to develop.

Although the experimental imformation obtained from this investigation definitely points out (at least in the case of filter paper and paraffin wax) that the classical limit is not only approached but actually reached, and the "excess scattering" vanishes at a very short wavelength, yet it is not possible, without further experiments under more favourable circumstances, to say with certainty what happens after that - whether the classical limit is the ultimate limit, or /

or whether with still harder incident radiation, the curve proceeds still further below the classical limit - and if so, in what way? Do the different curves cross one another at the point marking the classical limit, or do they continue without intersection? Only further experiments can settle these points. They are important for an understanding of the fundamental mechanism of the phenomenon of scattering by electrons within the atom.

With a view to determining how the "excess scattering" in the forward direction varies from angle to angle, for the same incident radiation and for the same scatterer, experiments were performed with filter paper (70 sheets) irradiated with radiation of equivalent wavelength .49 Å.U. (excitation voltage 80 K.V.peak, filtered with .54 mm. Al.) In the following table, (12), the experimental ratios (corrected) for different angles together with the corresponding values of $(1 + \cos^2\phi)$, and also the values predicted by Dirac's theory, are recorded in different columns. The 5th and 6th columns respectively show the percentage excess of the experimental and Dirac's ratios over the simple classical value $(1 + \cos^2\phi)$.

Table 12 /

Table 12.

Angle ϕ	Exptl. I_ϕ/I_{90} (corrected)	$(1 + \cos^2\phi)$	Dirac I_ϕ/I_{90}	% Excess over $(1 + \cos^2\phi)$	
				Exptl. I_ϕ/I_{90}	Dirac I_ϕ/I_{90}
20°	3.13	1.88	2.14	66.5	13.8
30°	2.21	1.75	1.98	26.3	13.1
40°	1.78	1.59	1.78	11.9	11.9
60°	1.30	1.25	1.34	4.0	7.2
90°	1	1	1	0	0

The above results are better illustrated by the curves in fig. 8 P99, where % excesses of the experimental and Dirac's values over the value of $(1 + \cos^2\phi)$ have been plotted against the angle ϕ. The R.H.S. of the curve has been drawn from symmetry with the L.H.S. one, which was obtained from experiment. It is clearly seen how rapidly the experimental curve rises as the direction of the primary beam is approached. As investigation was impossible for angles less than 20°, the course of the curve there could not be explored, and consequently the minimum reported by Hewlett for very small angles could not be tested or studied.

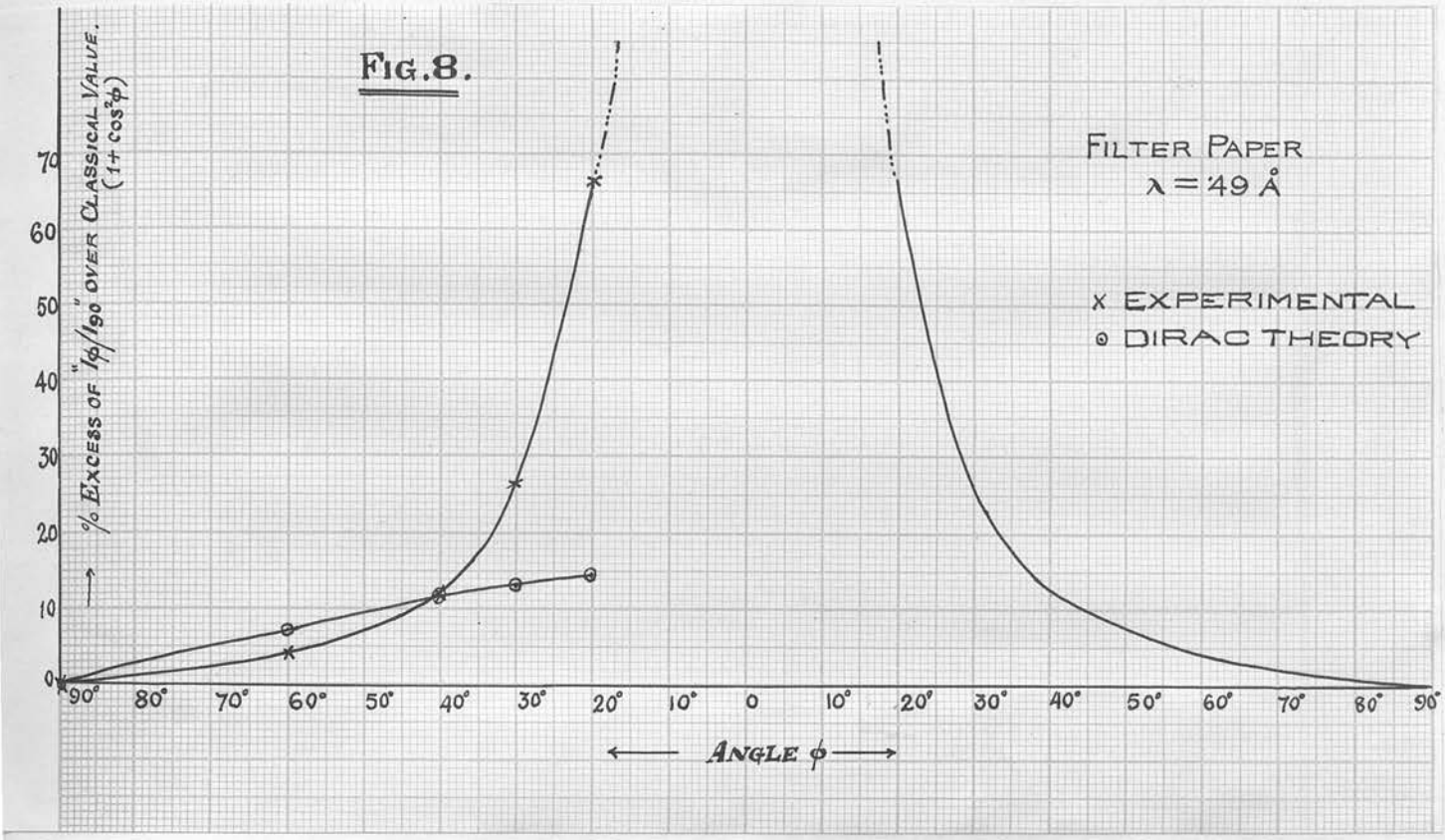
It is worth notice in this picture, that at a particular value of the angle ϕ - about 40° - the two curves intersect each other, showing the experimental value of the ratio there to /

to be coincident with Dirac's, whereas for angles smaller, the former is distinctly greater than the latter - the more so the smaller the angle. The curves are very close to each other through quite a big angular range, 40° to 90°, so that the difference between the Experimental and Dirac's values of the ratio "I_{ϕ}/I_{90}" there is very small - of the order of about 3 p.c. of the whole. But that it represents nothing more than a mere accident, for $\lambda = .49$ Å.U., can be shown from the following consideration. As the wavelength diminishes, the experimental curve moves down; whereas the Dirac's curve - as can be seen from the Dirac's equation - moves up, increasing the above discrepancy between the two. With an increase in the wavelength, again the experimental curve moves upwards and the Dirac's downward, and the values of the ratio are again divergent for very long waves. For very short wavelengths, Dirac's values of the ratio become greater than the experimental, while for very long wavelengths the reverse holds. It is in a limited region of medium wavelengths only, that the difference between the values of the ratio, experimental and Dirac's, becomes small, and that even for a short range of angles depending on the scattering substance, and included in the forward direction near $\phi = 90^{\circ}$. In addition, as different substances show different amounts of excess scattering for the same incident wavelength and the same angle, this limited region /

region of medium wavelengths is in all probability different and differently situated in the scale of wavelengths, for different scatterers.

Besides, it may also be noted in this curve (fig. 8) which has been drawn for filter paper and which may be regarded as typical of the series representing other scatterers also, that although the experimental values "I_{ϕ}/I_{90}" are in excess of the simple classical given by $(1 + \cos^2\phi)$, and the more so the smaller the angle, yet for a very narrow range of angles - here from about 65° to 90° - the experimental ratios tally satisfactorily with the simple classical. This angular range may be enlarged by the use of shorter incident waves. And the fact that, for a definite angle ϕ, the shorter the wavelength, the closer is the agreement with the simple classical result, has been already observed in the case of $\phi = 30^{\circ}$ and 20°, for different scatterers. Here also as before, the range of angles for which agreement holds, depends on the substance that scatters and on the wavelength of the incident radiation that is scattered.

Fig. 8 /



Let us now compare these results with those of other investigators in this field of research.

(1) As regards the dependence of the relative scattering "I_ϕ/I_{90}" on the material of the scatterer, Chylinski observed that at an effective wavelength of .23Å, aluminium yields a value of I_{30}/I_{90}, which is greater than that given by paraffin; and at an effective wavelength .19 Å, the value I_{30}/I_{90} (uncorrected) for lead is greater than for copper. Coven also obtained results of similar kind, using .32 Å (effective) for paraffin and aluminium, and .27 Å (effective) for copper and lead. These results, though mostly uncorrected, are in agreement with ours and support the general conclusion that /

that I_{ϕ}/I_{90} (when ϕ is less than 90°) increases with the atomic number of the scatterer.

(2) Turning to the angular distribution of the intensity of the scattered radiation we find that Coven reports good agreement with Dirac's theoretical values for paraffin wax at an effective wavelength of .32 Å, in the range of angles 60° to 120°, which lies partly in the forward and partly in the backward direction. But for angles lying between 60° and 30°, the experimental results were below the theoretical. This latter feature in the forward direction is explicable on the basis of the results indicated in fig. 8, P99. Coven's results are also in accord with those of Jauncey and Harvey, who found agreement with Dirac's theory near 90°, for a paraffin scatterer at $\lambda = .3$ Å approximately.

(3) Chylinski, on the contrary, working with a paraffin scatterer, observed a distinct excess over the Breit-Dirac theoretical value at an effective wavelength .23 Å, both in the forward and the backward direction (angular range 10° to 105°). For such a short wave and such a light scatterer - the conditions which produce little excess scattering - we would expect the difference between Experiment and Dirac theory in the forward direction to be in the opposite sense, as for such a short wavelength Dirac's values are high.

(4) Backhurst from experiments on the scattering by beryllium concludes that over a range of angles ϕ from 90° to say 40° there /

there is close agreement with Dirac's theory. For the angle 30^{o}, however, there is definitely an excess though not a large one. The smallness of the excess is, of course due to the low value of the atomic number (4).

(5) A comparison of our results with those of Khubchandani, tabulated on page 15 shows that, in the forward direction, they are in conformity with each other, at least in a qualitative manner. They (together with Owen's results) agree in that the excess scattering undergoes a progressive reduction with the hardening of the incident radiation. The only difference is that, while his work was confined to more or less long and medium wavelengths, the present investigation extended far beyond into the region of short waves where also, the same phenomenon was observed to hold till the simple classical limit was reached.

As regards the effect of thickness of the radiator on the value of the relative scattered intensity, experiments were performed, both in the forward and in the backward directions, with 24 sheets of filter paper, with incident radiations defined by equivalent wavelengths .49 Å and .44 Å, and the results compared with the corresponding ones, for 70 sheets of filter paper. No appreciable difference can be said to have been detected between the two cases after correction, as will be seen from the table below.

Table 13 /

Table 13.

Equiv. λ	No. of sheets	Uncor. I_{30}/I_{90}	Uncor. I_{150}/I_{90}	Corrected I_{30}/I_{90}	Corrected I_{150}/I_{90}
.49 A	70	2.24	1.75	2.21	1.56
(80 K.V. filt. .54 mm. Al.)	24	2.29	1.73	2.23	1.57
.44 A	70	2.165	1.73	2.16	1.51
(100 K.V. filt. .54 mm. Al.)	24	2.19	1.75	2.14	1.53

It should be remarked here that the stray effect in the forward direction was found to be greater for the thinner radiator than for the thicker, which accounts for the uncorrected "I_{30}/I_{90}" being somewhat greater for the thinner than for the thicker.

General consideration.

The Breit-Dirac theory of scattering has a restricted application in the sense that it takes no account of the existence of any coherent scattered radiation from different electrons within the atom or from different neighbouring atoms. It should, therefore, be more rigorously applicable to the case of monatomic perfect gases - preferably the light ones - than to the case of solids or liquids, where the configuration of the electrons within the atom, the configuration of the atoms within the molecule, and any special orientation of the molecules themselves, may co-operate with one another in complicating the real issue, viz the fundamental mechanism which controls the scattering phenomenon. The extent of such complications will, in general, be a function of (1) the atomic number of the scatterer, (2) the angle of scattering, (3) the wavelength of the radiation, as well as (4) atomic or molecular configurations.

A direct consequence of the above-mentioned coherence is, under suitable circumstances, a possibility of interference in the optical sense, between the rays scattered from different sources. Under the conditions of wavelength and /

and angle favourable to coherent scattering, * Debye and Scherrer (Gottinger Nachrichten, 1916) obtained for solid powders clear evidence of patterns arising from such interference on the photographic plate. Hewlett also demonstrated these interference effects for some liquids. A study of the maxima and minima of interference gave conclusive indication of the crystal structure of the solids and liquids in question, the seat of coherent waves being in these cases the atoms arranged in regular crystal lattices.

Similarly, the scattered rays from different electrons within /

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* Most recently Thomer (Phys. Zeits. 38. pp 48 - 57. Jan 15, 1937) has tested Debye's theory for angular distribution of scattered X-rays in the case of Neon-like molecules such as Ne, H_2O, NH_3, CH_4 (each having an electron number 10) and also C_6H_6, using a monochromatic radiation (K_α-radiation of Cu). He has shown that after applying necessary corrections and giving appropriate values to the "screening constant" the experimental results can be made to fit in satisfactorily with the theory.

within the same atom may also interfere, and in the case of an interference pattern, it is well known that the intensity of the maximum of any order diminishes with the increase in the order, i.e. in the obliquity of the rays concerned.

In the backward direction, particularly for an angle as big as 150°, the effect of interference is practically absent, and as such, this angle forms a suitable direction for testing any theory, free from most of the complications. And along this direction, the Breit-Dirac theory, in general, has been found valid for all radiators, and for all radiations (except probably for the shortest $\lambda = .225$ Å.U.) used in this investigation. This corroborates Backhurst's results, "that within certain limitations of angles and wavelengths the scattering from solids, composed of atoms of any kind and consisting, as in the normal state, of aggregates of small crystals or amorphous material, is given within a few per cent by equations based on Wave Mechanics, for the scattering from gases" - (Backhurst, Phil. Mag. Feb. 1934).

In the forward direction, on the other hand, the effect of superposition of scattered waves agreeing in phase, is calculated to be great. Accordingly, if the observed excess scattering owes its origin to this superposition, then it should be of a greater magnitude the closer the agreement in phase. The effect of phase agreement becomes more marked as (1) the wavelength becomes larger

(2) the /

(2) the scattering angle Ø becomes smaller

(3) the distance between the interfering sources becomes smaller.

The excess relative intensity should increase also, with the atomic number of the scattering substance, for two reasons; - the increased number of interfering sources (electrons), and secondly, the closer packing of the electrons inside the atom, assuming the size of the atom to remain, to a first approximation, constant, for all atomic numbers.

Let us now examine on the basis of the above tests, how far the results of experiment in the forward direction are in conformity with the idea of interference. First of all, the very appearance of the curve (experimental) in fig. 8 is suggestive of the central band of an interference pattern. Secondly, we have already seen that for the same radiator, the excess scattering is larger the larger the wavelength and the smaller the angle. Also, for the same wavelength and the same angle, the excess scattering increases with atomic number: the excess scattering from sulphur ($N = 16$) is greater than that from aluminium ($N = 13$) and the excess scattering from aluminium is, again, greater than from carbon ($N = 6$). Although no definite atomic number, in the ordinary sense, can be assigned to paraffin wax, and filter paper, yet the fact that filter paper yields a greater excess scattering than carbon, and paraffin wax less, is in general /

general agreement with the idea. For, the oxygen in filter paper and hydrogen in paraffin would contribute to the excess in precisely this way. The observed results thus fully endorse the idea of interference.

In addition, owing to the greater proportion of longer wavelengths in the beam scattered in the forward direction ($\phi < 90^{\circ}$), one would naturally expect this beam to be relatively softer (or more absorbable) than that scattered along $\phi = 90^{\circ}$, assuming, of course, there is no other effect to alter the relative absorbabilities. The effect of interference to which the excess scattering is due, is thus, in the case of heterogeneous beams, opposite in sense to the Compton effect which tends to produce the reverse result. This is the more precise specification of the "superposed effect" to which we made reference earlier in connection with an attempt to explain the apparently anomalous behaviour of aluminium and sulphur, so far as the absorbability of rays scattered by them was concerned.

It now remains to determine, for the forward direction, whether the scattered intensity from individual electrons follows the classical law of Sir J. J. Thomson, or the quantum law as predicted by Breit and Dirac. The task seems to be quite a difficult one, in view of the fact that the simple intensity distribution for each separate electron /

electron has been masked by less simple interference effects. Nevertheless, the right procedure in such an attempt would be to study the results for short wavelengths; since most of the complications arising from interference effects are calculated to disappear in that case.

Looking from this angle, the experimental fact that filter paper and paraffin have almost attained the Thomson value of I_{30}/I_{90} for the shortest wavelength .225 Å - the value being about 30% lower than the corresponding Breit-Dirac value - suggests a complete departure from the theory proposed by Breit-Dirac on the basis of quantum mechanics, so far as the forward direction is concerned. Notwithstanding the chance agreement with the quantum theory - in the forward direction - under certain limitations of atomic number, wavelength and angle, as has been explained earlier, the experimental evidence remains in favour of the classical theory in the forward direction, unless, of course, the so-called classical limit of I_{ϕ}/I_{90} is ~~not~~ passed as a result of further experiments with still shorter wavelengths.

General Conclusions.

An attempt has been made in this paper to study the intensity distribution of X-radiation scattered from various substances. Employing heterogeneous primary radiations extending over a wide range of wavelengths and making proper corrections, the ratio I_ϕ/I_{90} (where I_ϕ means the intensity of the scattered radiation in a direction making an angle ϕ with the primary beam) has been determined experimentally for angles $\phi = 20^\circ$, 30°, 40°, 60° and 150°. The information obtained, as far as these experiments go, may be summarised briefly as follows.

1. In the backward direction, $\phi = 150^\circ$

   (a) the experimental ratio I_ϕ/I_{90}, though approximating to the value $(1 + \cos^2\phi)$ for long waves, is invariably less than that given by this simple law.

   (b) For a particular wavelength, the deficiency of the experimental ratio I_ϕ/I_{90} below $(1 + \cos^2\phi)$ is independent of the nature of the scattering material (within experimental error).

   (c) The deficiency of the experimental ratio I_ϕ/I_{90} below $(1 + \cos^2\phi)$ increases as the wavelength decreases.

2. In /

2. In the forward direction

(a) The ratio I_{ϕ}/I_{90}, though approximating to the value $(1 + \cos^2\phi)$ for short waves, is invariably greater than that given by this simple law.

(b) For a particular wavelength, the excess of the experimental ratio I_{ϕ}/I_{90} over $(1 + \cos^2\phi)$, depends largely on the nature of the scattering material, increasing with an increase in the atomic number (or an "average atomic number", as the case may be) of the scattering material.

(c) For a particular scatterer, the excess of the experimental ratio I_{ϕ}/I_{90} over $(1 + \cos^2\phi)$ decreases as the wavelength decreases.

(d) For a particular wavelength and a particular scatterer, the excess of the experimental ratio I_{ϕ}/I_{90} over $(1 + \cos^2\phi)$ decreases as the angle ϕ increases.

In the backward direction, the intensity of scattering from different substances agrees very closely with that given by the quantum theory of Dirac, whereas in the forward direction, the observed scattering is complicated by interference effects and may be explained on the classical theory.

In /

In conclusion, I wish to take this opportunity of expressing my deep sense of gratitude to Professor C. G. Barkla, F.R.S., Nobel Laureate, for his kind and constant supervision of this research. My thanks are also due to the members of the staff who helped me in many ways during this work.

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Crystal Structure of Hexaethylbenzene $C_6(C_2H_5)_6$

By

H. K. Pal and A. C. Guha

1935

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Crystal Structure of Hexaethylbenzene $C_6(C_2H_5)_6$.

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1. Introduction.

The crystal structure of hexamethylbenzene $C_6(CH_3)_6$, analysed by Mrs. Lonsdale[1]), offers many points of interest. It was the first compound in which the plane regular hexagonal structure of the benzene ring was definitely established. The aliphatic carbon atoms of the molecule also lie in the plane of the benzene ring, being attached to their respective carbon atoms in the ring radially. Further, hexamethylbenzene is one of the very few crystals that contain only one molecule in the unit cell.

It would be of interest to study the structure of the homologous compound hexaethylbenzene, $C_6(C_2H_5)_6$. A short account is given in this paper of some goniometric and X-ray measurements on this crystal.

2. Goniometric Measurements.

Crystals are easily grown out of solutions in acetone or benzene. They are found to belong to the triclinic system, having the axial ratios and angles

$$a : b : c = 1.004 : 1 : 0.610;$$
$$\alpha = 58°\,5',\ \beta = 103°\,54',\ \gamma = 123°\,43'.$$

The following faces are well-developed:— {010}, {011}, {001}, {100}, {1$\bar{1}$0}, and less frequently {$\bar{1}$11}. The crystals occur usually in the form of thick plates parallel to {010}, and sometimes as columnar crystals elongated parallel to the c axis. The angles between the different faces as measured (with a single-circle goniometer), and as calculated from the above axial angles and ratios, are given in the following Table.

Faces and Angles.

	Measured	Calculated		Measured	Calculated
(100) : (010)	58° 38′	58° 36′	(001) : (0$\bar{1}$0)	60 37	60 34
(010) : ($\bar{1}$10)	53 36	53 37	($\bar{1}$10) : (011)	— —	92 23
($\bar{1}$10) : ($\bar{1}$00)	67 46	67 45	(011) : (1$\bar{1}$0)	— —	87 37
(100) : (011)	71 12	70 56	($\bar{1}$10) : ($\bar{1}$11)	77 27	77 51
(011) : ($\bar{1}$11)	39 55	40 21	($\bar{1}$11) : (001)	39 56	39 32
($\bar{1}$11) : ($\bar{1}$00)	68 53	68 43	(001) : (1$\bar{1}$0)	62 37	62 37
(010) : (011)	75 38	75 48	(010) : ($\bar{1}$11)	— —	96 29
(011) : (001)	43 44	43 38	(100): (001)	— —	94 31

1) Lonsdale, K., Proc. Roy. Soc. London (A) **123** (1929) 494; Trans. Farad. Soc. **25** (1929) 352.

3. X-Ray Measurements.

X-ray rotation photographs were taken about (1) the c axis, (2) the b axis, and (3) the intersection of (001) and ($\bar{1}10$) planes; the spacings obtained were 6.10, 9.84 and 9.31 Å respectively. They give for the dimensions of the unit cell

$$a = 9.90, \quad b = 9.84, \quad c = 6.10 \text{ Å},$$

the axial angles α, β, γ having been assumed to be the same as determined goniometrically. These dimensions correspond to the ratios

$$a : b : c = 1.006 : 1 : 0.620,$$

which agree with the values obtained in the previous section.

The volume of the unit cell is equal to $abc \sin\beta \sin\gamma \sin\delta$, where

$$\sin\frac{\delta}{2} = \sqrt{\frac{\sin(\alpha - \beta + \gamma)/2 \cdot \sin(\alpha + \beta - \gamma)/2}{\sin\beta \sin\gamma}},$$

and is found to be $418 \cdot 10^{-24}$ c. c. Taking the density of the crystal to be 0.94, we obtain for the mass of the unit cell $393 \cdot 10^{-24}$ gms., as compared with the molecular weight of $C_6(C_2H_5)_6$, $406 \cdot 10^{-24}$ gms. There is thus only one molecule in the unit cell.

The crystal has a centre of symmetry, and it is probable that the molecule also has the same symmetry.

4. Comparison with Hexamethylbenzene.

Comparing the above data with those for the corresponding methyl derivative $C_6(CH_3)_6$,

$a = 9.010$ Å	$a = 44° 27'$
$b = 8.926$	$\beta = 116\ 43$
$c = 5.344$	$\gamma = 119\ 34$

we find a strong resemblance between the two crystals. Both of them are triclinic, with one molecule in the unit cell. For both the crystals the a and the b axes are nearly equal, and are inclined at about 120°, and there is a pseudo-hexagonal symmetry in the (001) plane (this is brought out very clearly in the stereographic projection, in which all the angles at the pole (001) are nearly 60°). This pseudo-hexagonal symmetry in the (001) plane suggests that, as in hexamethyl benzene, in hexaethylbenzene also the benzene rings are nearly parallel to (001). Regarding the size of the unit cell we find that, as we proceed from hexamethylbenzene to hexaethylbenzene, (1) the a and b axes have increased from 9.0 to

9.9 Å, i. e., by 0.9 Å, which is much less than the *C-C* distance, viz., 1.54 Å; (2) the (001) spacing has increased from 3.69 Å in the methyl compound to 5.16 Å in the ethyl compound. This suggests that some of the carbon atoms in hexaethylbenzene, presumably the outermost aliphatic ones, do not lie in the (001) plane, but are considerably displaced from it.

The detailed X-ray analysis of the structure of this crystal will be published in due course.

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